3rd year secondary

Calculus

booklets 2017

إعداد فريق العمل

أ/جمال الشاهد	موجه اول المدارس الرسميه للغات بمحافظه الجيزه
أ/محمد يس جمعه	موجه رياضيات
أ/محمد على السعيد	موجه رياضيات
أ/سيد محمد عبد العزيز	موجه رياضيات
أ/بسيونى ابراهيم	كبير معلمين (مدرسه الاورمان)
أ/محمد على قاسم	معلم اول .أ (مدرسه هضبه الاهرام)

مع خاص التمنيات للطلبه بالنجاح و التوفيق

Guide Answers

Answer the following questions 20 questions **Model Answer**

Q(1) If $F(X)=\frac{X}{X-2}$, then $F'''(3)$ equals……………………

① -36 ② -12

③ 6 ④ 4

$$F'(X)=\frac{X-2-X}{(X-2)^2}=-2(X-2)^{-2}\quad \therefore F''(X)=4(X-2)^{-3}\quad \therefore F'''(X)=-12(X-2)^{-4}=-12$$

Q(2) If $F(X)=\ln(X^2+1)^2+e^{\sin X}$ then $F'(0)=$ ……..

① 1+e ② 1

③ e ④ 0

$$F'(X)=\frac{4X}{X^2+1}+\cos Xe^{\sin X}\quad \therefore F'(0)=\frac{4\times 0}{0^2+1}+\cos 0\times e^{\sin 0}=1$$

Q(3) If $F(X)=X^X$ then $F'(1)=$……………….

① e ② 1

③ -1 ④ 0

$$Y=X^X\quad \therefore \ln Y=X\ln X\quad \therefore \frac{Y'}{Y}=\ln X+1\quad \therefore Y'=X^X\ln X+X^X=1$$

Q(4) A body moves on the curve $Y^2=X^3$ If $\frac{dX}{dt}=\frac{1}{2}$ unit/sec when Y=-1 then $\frac{dY}{dt}$ at this moment equals……… unit/s

① $-\frac{3}{4}$ ② $-\frac{3}{8}$

③ $\frac{3}{4}$ ④ $\frac{3}{2}$

$$2Y\frac{dY}{dt}=3X^2\frac{dX}{dt}\quad Y=-1\therefore X=1\quad \therefore 2(-1)\times\frac{dY}{dt}=3(1)^2\times\frac{1}{2}\quad \therefore \frac{dY}{dt}=-\frac{3}{4}$$

Q(5) The slope of tangent to the curve $Y = \ln\left(2 - \sqrt{2}\cos X\right)$ at $X = \frac{\pi}{4}$ equals ……

① 1 ② 2

③ -1 ④ -2

$$y' = \frac{\sqrt{2}\sin X}{2 - \sqrt{2}\cos X} \quad \text{at} X = \frac{\pi}{4} \quad \therefore y' = \frac{\sqrt{2}\sin 45^\circ}{2 - \sqrt{2}\cos 45^\circ} = 1$$

Q(6) If $Y = \sqrt{e^{2X} + \ln(X+1)}$ then $\frac{dY}{dX}$ =……. when X=0

① $\frac{3}{2}$ ② $\frac{1}{2}$

③ 0 ④ 1

$$\frac{dY}{dX} = \frac{1}{2}\left(e^{2X} + \ln(X+1)\right)^{-\frac{1}{2}} \times \left(2e^{2X} + \frac{1}{X+1}\right) \text{ at } X = 0 \quad \frac{dY}{dX} = \frac{1}{2}\left(e^{0} + \ln 1\right)^{-\frac{1}{2}} \times \left(2e^{0} + 1\right) = \frac{3}{2}$$

Q(7) $\lim\limits_{h\to 0} \frac{\cos\left(\frac{\pi}{3} + h\right) - \frac{1}{2}}{h}$

① $\frac{1}{2}$ ② $\frac{\sqrt{3}}{2}$

③ $-\frac{1}{2}$ ④ $-\frac{\sqrt{3}}{2}$

$$\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{3} + h\right) - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{3} + h\right) - \cos\frac{\pi}{3}}{h} = \frac{d}{dX}(\cos X) = -\sin X = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

Q(8) $\lim\limits_{X\to\infty}\left(1 - \frac{1}{X}\right)^{2X} =$

① e^{2} ② 1

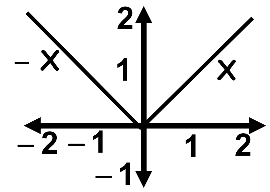
③ e^{-2} ④ 0

$$\text{let } -\frac{1}{X} = Y \quad \therefore X = -\frac{1}{Y} \quad \therefore \lim_{Y\to 0}(1+Y)^{-\frac{2}{Y}} = \lim_{Y\to 0}\left[(1+Y)^{\frac{1}{Y}}\right]^{-2} = e^{-2}$$

Q(9) If $F(X) = |X|$ then $\int_{-2}^{2} f(X)dX = \dots\dots$

① -1 ② 2

③ 0 ④ 4



$$2\int_0^2 XdX = 2\left[\frac{X^2}{2}\right]_0^2 = 2\times\left[\frac{4}{2}-0\right] = 4$$

Q(10) $\int_e^{2e} \frac{1}{X}dX = \dots\dots$

① 1 ② e

③ 0 ④ ln2

$$\int_e^{2e} \frac{1}{X}dX = [\ln X]_e^{2e} = \ln 2e - \ln e = \ln 2$$

Q(11) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin X|dX =$

① 0 ② 2

③ 1 ④ 4

$$A = 2\int_0^{\frac{\pi}{2}} \sin X\, dX = 2[-\cos X]_0^{\frac{\pi}{2}} = 2\left[\left(-\cos\frac{\pi}{2}\right)-(-\cos 0)\right] = 2 - -1 = 2$$

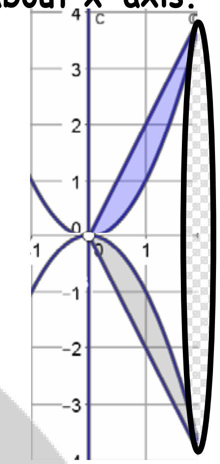
Q(12) $\int_1^{e^2} \frac{\ln^2 X}{X}\, dX =$

① $\frac{7}{3e^2}$ ② $\frac{7}{3}$

③ $\frac{8}{3}$ ④ $\frac{4}{e^2}$

$$\int_1^{e^2} \frac{\ln^2 X}{X}\, dX = \left[\frac{(\ln X)^3}{3}\right]_1^{e^2} = \frac{(\ln e^2)^3}{3} - \frac{(\ln 1)^3}{3} = \frac{(2\ln e)^3}{3} = \frac{8}{3}$$

Q(13) Find the volume of the solid generated by revolving the region bounded by the curve $y = X^2$ and the straight line $y = 2X$ a complete revolution about x-axis.



$X^2 = 2X \quad \therefore X^2 - 2X = 0 \quad \therefore X(X-2) = 0$

$X = 0$ or $X = 2$

$$V = \pi\int_0^2 \left(Y_1^2 - Y_2^2\right)dX = \pi\int_0^2 \left(4X^2 - X^4\right)dX$$

$$= \pi\left[\frac{4}{3}X^3 - \frac{1}{5}X^5\right]_0^2 = \pi\left(\frac{23}{3} - \frac{32}{5}\right) = \frac{64}{15}\pi$$

Q(14) If $Y = ae^{\frac{b}{X}}$ <u>prove that</u> : $XYY'' + 2YY' - X(Y')^2 = 0$

$\ln Y = \ln a + \frac{b}{X} \qquad \times X \qquad \therefore X\ln Y = X\ln a + b$

$\ln Y + X\frac{Y'}{Y} = \ln a$

$\frac{Y'}{Y} + \frac{Y'}{Y} + X\frac{Y''Y - Y'^2}{Y^2} = 0 \qquad \times Y^2$

$Y'Y + Y'Y + XYY'' - XY'^2 = 0$

$\therefore XYY'' + 2YY' - X(Y')^2 = 0$

Q(15)Find the absolute maximum and absolute minimum value of the function $Y = \frac{X}{1+X^2}$ **in the interval** $[-1,2]$

$$Y' = \frac{1(1+X^2) - 2X \times X}{(1+X^2)^2} = \frac{1+X^2-2X^2}{(1+X^2)^2} = \frac{1-X^2}{(1+X^2)^2} = 0$$

$1 - X^2 = 0 \quad \therefore X = 1$ or $X = -1$

$F(1) = \frac{1}{2} \quad , F(-1) = -\frac{1}{2} \quad , F(2) = \frac{2}{1+4} = \frac{2}{5}$

$A.\text{maximum} = \frac{1}{2}$

$A.\text{minimum} = -\frac{1}{2}$

Q(16) Using on of the integration methods to find $\int (\ln X)^2\, dX$

$$= X(\ln X)^2 - \int 2\ln X dX$$
$$= XX(\ln X)^2 - 2\left(X\ln X - \int X \times \frac{1}{X} dX\right)$$
$$= X(\ln X)^2 - 2(X\ln X - X)$$
$$= X\left[(\ln X)^2 - 2\ln X + 2\right] + C$$

D	I
$(\ln X)^2$	dX
$\frac{2}{X}\ln X$	X

D	I
$\ln X$	dX
$\frac{1}{X}$	X

Q(17) Using one of the integration techniques to find $\int \frac{X + \sin X}{1 + \cos X}\, dX$

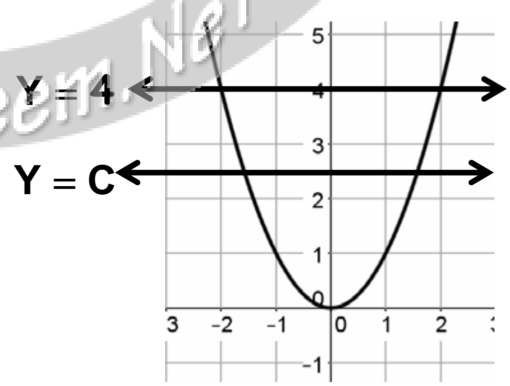
$$= \int \frac{X}{1+\cos X} dX + \int \frac{\sin X}{1 + \cos X} dX$$
$$= \int \frac{X}{2\cos^2 \frac{X}{2}} dX + \int \frac{\sin X}{2\cos^2 \frac{X}{2}} dX = \frac{1}{2}\int X \sec^2 \frac{1}{2} X dX + \int \frac{2\sin\frac{X}{2}\cos\frac{X}{2}}{2\cos^2\frac{X}{2}} dX =$$
$$\int X \sec^2 \frac{1}{2} X dX + \int \frac{\sin\frac{X}{2}}{\cos\frac{X}{2}} dX = \frac{1}{2}\int X \sec^2 \frac{1}{2} X dX + \int \tan\frac{X}{2} dX =$$
$$\frac{1}{2}\left(2X\tan\frac{1}{2}X - \int 2\tan\frac{1}{2}X dX\right) + \int \tan\frac{X}{2} dX = X\tan\frac{X}{2} + C$$

D	I
X	$\sec^2\frac{1}{2}X$
1	$2\tan\frac{1}{2}X$

Q(18) If the line $Y = C$ divide the area between $Y = X^2$ and Y=4 into two equal parts then find the value of C

$X^2 = 4 \quad \therefore X = \pm 2$

$$\int_{-2}^{2}(4 - X^2)dX = \left[4X - \frac{X^3}{3}\right]_{-2}^{2} = \frac{32}{3}$$
$$\therefore \frac{32}{3} \div 2 = \frac{32}{6}$$
$$\therefore 2\int_0^{\sqrt{C}} (C - X^2)dX = \frac{32}{6}$$
$$\therefore 2\left[CX - \frac{X^3}{3}\right]_0^{\sqrt{C}} = \frac{32}{6} \therefore 2\left(C\sqrt{C} - \frac{C\sqrt{C}}{3}\right) = \frac{32}{6} \therefore C = \sqrt[3]{16}$$



Q(19) If the slope of the tangent equals $\frac{\sqrt{e^Y}}{1-\cos^2 X}$ and the curve passing through the point ($\frac{\pi}{4}$,0) find the equation of the curve

$$\frac{dY}{dX}=\frac{\sqrt{e^Y}}{1-\cos^2 X} \quad \therefore \int\frac{dY}{\sqrt{e^Y}}=\int\frac{dX}{1-\cos^2 X}$$

$$\therefore \int e^{-\frac{1}{2}Y}dY=\int \csc^2 X dX \therefore -2e^{-\frac{1}{2}Y}=-\cot X+C \quad \text{at} \left(\frac{\pi}{4},0\right)$$

$$\therefore -2=-\cot 45^\circ+C \quad \therefore C=-1$$

$$\therefore -2e^{-\frac{1}{2}Y}=-\cot X-1 \quad \therefore 2e^{-\frac{1}{2}Y}=\cot X+1 \quad \text{take ln both sides}$$

$$2\ln e^{-\frac{1}{2}Y}=\ln(\cot X+1) \quad \therefore Y=-\ln(\cot X+1)$$

Q(20) In the given figure : $F(X)=X^2$ find the greatest area of rectangle

Equation of $\overleftrightarrow{AC}$ is $X+Y=5 \quad \therefore Y=5-X$

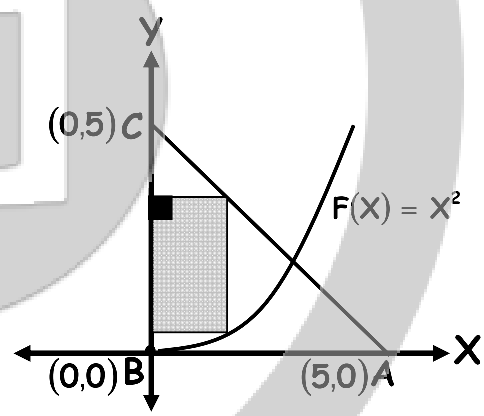
$A=X(5-X-X^2)$

$A=5X-X^2-X^3$

$\frac{dA}{dX}=5-2X-3X^2$

$\therefore X=1 \therefore \left.\frac{d^2A}{dX^2}\right|_{X=1}=-2-6X<0$

$\therefore A=1(5-1-1)=3$ square units



☆**Find the two equations of the tangent and the normal to the curve $X^2+3XY+Y^2+1=0$ at point A (- 1, 1). If they intersect x-axis at the two points B and C, calculate the area of the triangle A B C in squared units**

$2X+3\left(Y+X\frac{dY}{dX}\right)+2Y\frac{dY}{dX}=0$ at (-1,1) $\therefore \frac{dY}{dX}=1$

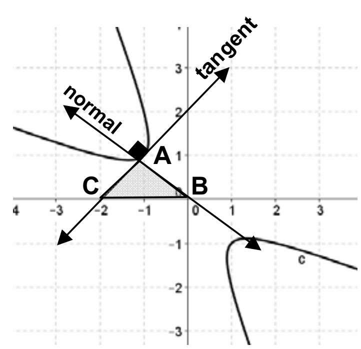
Equation of tangent $\frac{Y-1}{X+1}=1 \quad \therefore Y-X=2$

Equation of normal $\frac{Y-1}{X+1}=-1 \quad \therefore Y+X=0$

Tangent interest the X-axis at $Y-X=2$ put Y=0 $\therefore X=-2$

Tangent interest the X-axis at $Y+X=0$ put Y=0 $\therefore X=0$

Area of Δ ABC equal $\frac{1}{2}\times 2\times 1=1$ square units



Answer the following questions 20 questions **Model Answer**

Q(1) If $Y = \sin 2X \cos 2X$, then $Y''\left(\frac{\pi}{3}\right)$ equals :……….

① -4 ② 0

③ $4\sqrt{3}$ ④ 8

$Y = \frac{1}{2}\sin 4X \quad Y' = 2\cos 4X \quad \therefore Y'' = -8\sin 4X \quad -8\sin 4\times 60 = 4\sqrt{3}$

Q(2) If $F(X) = Xe^{-2X}$ then $F'\left(\frac{1}{2}\right)$ =……….

① 0 ② e

③ e^2 ④ e^{-1}

$F'(X) = e^{-2X} - 2Xe^{-2X} \quad \therefore F'\left(\frac{1}{2}\right) = e^{-1} - e^{-1} = 0$

Q(3) If $Y = X^2 \ln\frac{X}{a}$ then $\frac{d^3Y}{dX^3}$ =………. at X=4

① 1 ② 2

③ 4 ④ $\frac{1}{2}$

$\frac{dY}{dX} = 2X\ln\frac{X}{a} + \left(\frac{a}{X}\times\frac{1}{a}\right)X^2 = 2X\ln\frac{X}{a} + X$

$\frac{d^2Y}{dX^2} = 2\left(\ln\frac{X}{a} + \frac{a}{X}\times\frac{1}{a}X\right) + 1 = 2\left(\ln\frac{X}{a} + 1\right) \quad \therefore \frac{d^3Y}{dX^3} = 2\left(\frac{a}{X}\times\frac{1}{a}\right) = \frac{2}{X} = \frac{2}{4} = \frac{1}{2}$

Q(4) The sides of a right triangle with legs X and Y and hypotenuse Z increase in such a way that $\frac{dZ}{dt} = 1$ and $\frac{dX}{dt} = 3\frac{dY}{dt}$ at the instant when X=4 and Y=3 what is $\frac{dX}{dt}$

① 1 ② 5

③ 2 ④ $\frac{1}{3}$

$X^2 + Y^2 = Z^2 \quad \therefore 2X\frac{dX}{dt} + 2Y\frac{dY}{dt} = 2Z\frac{dZ}{dt} \quad \therefore X\frac{dX}{dt} + Y\frac{dY}{dt} = Z\frac{dZ}{dt}$

$4\frac{dX}{dt} + 3\times\frac{1}{3}\frac{dX}{dt} = 5\times 1 \quad \therefore 5\frac{dX}{dt} = 5 \quad \therefore \frac{dX}{dt} = 1$

Q(5) Slope of tangent of $Y = 1 + \sqrt{2}\csc X + \cot X$ At $\left(\frac{\pi}{4}, 4\right)$

① $-2+\sqrt{2}$ ② -4

③ 2 ④ -1

$\frac{dY}{dX} = -\sqrt{2}\csc X \cot X - \csc^2 X$ at $X = \frac{\pi}{4}$ $= -\sqrt{2}\times\sqrt{2}\times 1 - \left(\sqrt{2}\right)^2 = -2-2 = -4$

Q(6) If The line Y=X is tangent to the curve $Y = X^2 + K$ then K=……

① 2 ① 2

③ $\frac{1}{4}$ ③ $\frac{1}{4}$

$2X = 1 \therefore X = \frac{1}{2} \therefore Y = \frac{1}{2} \therefore \frac{1}{2} = \left(\frac{1}{2}\right)^2 + K \therefore K = \frac{1}{4}$

Q(7) $\lim_{h\to 0} \frac{\tan 2\left(\frac{\pi}{8}+h\right) - \tan\left(\frac{\pi}{4}\right)}{h} =$

① 2 ② 4

③ $3\sqrt{2}$ ④ $2\sqrt{3}$

$= F'(\tan 2X) = 2\sec^2 2X$ at $X = \frac{\pi}{8}$ $\therefore F'\left(\frac{\pi}{8}\right) = 2\sec^2(45°) = 4$

Q(8) If $X = t^3 - t$ and $Y = \sqrt{3t+1}$ then $\frac{dY}{dX}$ at t=1 is

① $\frac{1}{8}$ ② $\frac{8}{3}$

③ $\frac{3}{8}$ ④ $\frac{3}{4}$

$\frac{dX}{dt} = 3t^2 - 1$ $\frac{dY}{dt} = \frac{3}{2\sqrt{3t+1}}$ $\therefore \frac{dY}{dX} = \frac{3}{2\sqrt{3t+1}} \times \frac{1}{3t^2-1} = \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$

Q(9) If the function f is continuous and even on R and $\int_2^4 f(X)dX = 7$ and $\int_4^2 g(X)dX = 2$ then $\int_2^4 [2F(X) - 3g(X) - 5]dX = ...$

① -18 ② -8

③ 10 ④ 14

$\int_2^4 g(X)dX = -2 \quad \therefore 2\times 7 - 3\times -2 - [5X]_2^4 = 14 + 6 - (20 - 10) = 10$

Q(10) If $F'(X) = \frac{2}{X}$ and $F(\sqrt{e}) = 5$ then $F(e) = ...$

① 5 ② $\ln 25$

③ 6 ④ $\ln 5$

$F(X) = 2\ln X + C \therefore 5 = 2\ln e^{\frac{1}{2}} + c \quad \therefore 5 = 1 + C \quad \therefore c = 4 \quad \therefore F(X) = 2\ln X + 4$
$\therefore F(e) = 2\ln e + 4 = 6$

Q(11) $\int_0^2 [\, X^2 - |X - 1| \,] dX = ...$

① 0 ② $\frac{5}{3}$

③ $\frac{7}{3}$ ④ 1

$= \int_0^1 [X^2 - (1 - X)]dX + \int_1^2 [X^2 - (X - 1)]dX = \frac{5}{3}$

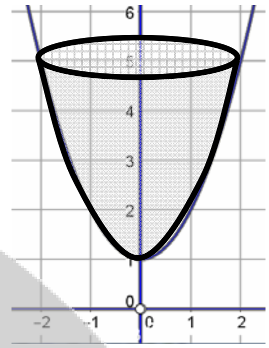
Q(12) If $\int_0^3 n(X + 1)^{n-1} dX = 15$ then the value of n=.....

① 1 ② 2

③ 4 ④ 3

$[(X + 1)^n]_0^3 = 15 \quad \therefore 4^n - 1^n = 15 \quad \therefore 4^n = 16 \quad \therefore n = 2$

Q(13) Find the volume of the solid generated by revolving the plane region bounded by the curve of the function $F(X) = X^2 + 1$, Y-axis and the two straight lines Y=5 a complete revolution about Y-axis



$$V = \pi\int_a^b X^2 dY$$

$$V = \pi\int_1^5 (Y - 1)dY =$$

$$= \pi\left[\frac{Y^2}{2} - Y\right]_1^5$$

$$= \pi\left[\left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)\right] = 8\pi$$

Q(14) If $Y - XY = \sin X$ prove that : $Y'' + Y = \frac{2Y'}{1-X}$

$Y' - (Y + XY') = \cos X \qquad \therefore Y' - Y - XY' = \cos X$

$Y'' - Y' - (Y' + XY'') = -\sin X \qquad \therefore Y'' - Y' - Y' - XY'' = -\sin X$

$Y''(1-X) = 2Y' - \sin X$

$Y'' = \frac{2Y' - \sin X}{1-X} = \frac{2Y'}{1-X} - \frac{\sin X}{1-X}$ adding Y to both sides

$Y'' + Y = \frac{2Y'}{1-X} - \frac{\sin X}{1-X} + Y \qquad \because Y = \frac{\sin X}{1-X}$

Q(15) If $Y = e^{-X}\sqrt{\frac{1+X}{1-X}}$, $-1 < X < 1$ prove that : $(1-X^2)Y' = X^2 Y$

$$\ln Y = \ln e^{-X}\left(\frac{1+X}{1-X}\right)^{\frac{1}{2}} = \ln e^{-X} + \ln\left(\frac{1+X}{1-X}\right)^{\frac{1}{2}} = -X + \frac{1}{2}(\ln(1+X) - \ln(1-X))$$

$$\therefore \frac{Y}{Y'} = -1 + \frac{1}{2}\left(\frac{1}{1+X} + \frac{1}{1-X}\right) = -1 - \frac{1}{2} \times \frac{1-X+1+X}{(1+X)(1-X)} = -1 + \frac{1}{1-X^2} = \frac{-1+X^2+1}{1-X^2}$$

$$\therefore \frac{Y}{Y'} = \frac{X^2}{1-X^2}$$

Q(16) find $\int \frac{3X+5}{e^{2X}}\,dX$

$$= -\frac{1}{2}e^{-2X}(3X+5) + \int \frac{3}{2}e^{-2X}$$
$$= -\frac{1}{2}e^{-2X}(3X+5) - \frac{3}{4}e^{-2X}$$

D	I
$3X+5$	e^{-2X}
3	$-\frac{1}{2}e^{-2X}$

Q(17) Using one of the integration techniques to find $\int \frac{2X+1}{e^{5X}}\,dX$

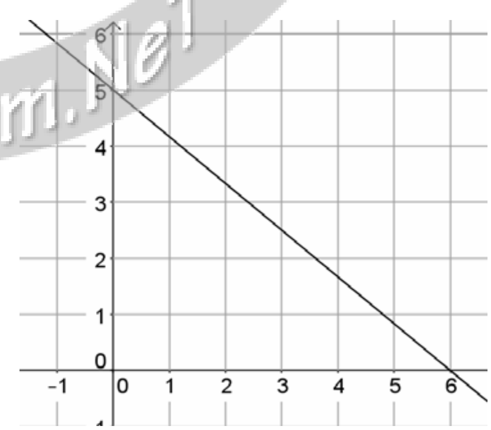
$$\int \frac{2X+1}{e^{5X}}\,dX = \int \left(2Xe^{-5X} + e^{-5X}\right)dX = \int \left(2Xe^{-5X}\right)dX + \int \left(e^{-5X}\right)dX$$

D	*I*
$2X$	e^{-5X}
2	$-\frac{1}{5}e^{-5X}$

$$\therefore -\frac{2}{5}Xe^{-5X} + \int \frac{2}{5}e^{-5X}dx = -\frac{2}{5}Xe^{-5X} - \frac{2}{25}e^{-5X}$$

$$\int \frac{2X+3}{e^{5X}}\,dX = -\frac{2}{5}Xe^{-5X} - \frac{2}{25}e^{-5X} - \frac{1}{5}e^{-5X}$$
$$-\frac{10}{25}Xe^{-5X} - \frac{2}{25}e^{-5X} - \frac{5}{25}e^{-5X} = -\frac{1}{25}e^{-5X}(10X+7)$$

Q(18) If you know that $F(0)=5$ Use the opposite figure which represent The graph of $F'(X)$ to find $F(6)$



$$\int_0^6 F'(X)dX = [F(X)]_0^6$$
$$F(6) - F(0) = \frac{1}{2}\times 6 \times 5 = 15$$
$$\therefore F(6) = 15 - 5 = 10$$

Q(19) **If $f(x) = 4 + \cot X - \sec^2 X$, find the equation of the normal to the curve of the function f at a point lying on the curve and its x-coordinate equals $\frac{\pi}{4}$**

When $X = \frac{\pi}{4}$ $\quad Y = 4 + \cot 45° - \sec^2 45° = 4 + 1 - 2 = 3$

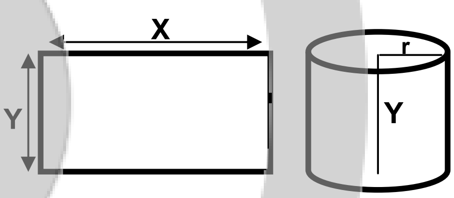
$F'(X) = -\csc^2 X - 2\sec^2 X \tan X$ at $X = 45°$

$F'\left(\frac{\pi}{4}\right) = -\csc^2 45° - 2\sec^2 45° \tan 45°$

$= -2 - 2 \times 2 = -6$

Equation of normal $\frac{Y - 3}{X - \frac{\pi}{4}} = \frac{1}{6}$

Q(20) **A rectangle of perimeter 30cm is revolved about one of its side to form a cylinder what is the maximum possible volume that could be generated**



$2X + 2Y = 30 \quad \therefore X + Y = 15 \quad \therefore Y = 15 - X$

$2\pi r = X \quad \therefore r = \frac{X}{2\pi}$

$V = \pi r^2 \times Y = \pi\left(\frac{X}{2\pi}\right)^2 \times (15 - X) = \frac{1}{4\pi}X^2(15 - X) = \frac{1}{4\pi}\left(15X^2 - X^3\right)$

$\frac{dV}{dX} = \frac{1}{4\pi}\left(30X - 3X^2\right) = 0 \quad \therefore 30X - 3X^2 = 0$

$\therefore 3X(10 - X) = 0 \quad \therefore X = 10 \quad \therefore Y = 15 - 10 = 5$

$V = \frac{1}{4\pi}\left(15 \times 10^2 - 10^3\right) = \frac{500}{4\pi}$

☆**If the tangent to the curve $Y = \sqrt{X}$ passes through the point (-1,0) find the equation of the tangent and the normal at the point of tangency**

Slope of tangent $\frac{1}{2}(X)^{-\frac{1}{2}}$ $\quad$ Slope of tangent $\frac{\sqrt{X} - 0}{X + 1}$

$\therefore \frac{1}{2}\frac{1}{\sqrt{X}} = \frac{\sqrt{X}}{X + 1} \quad \therefore 2X = X + 1 \quad \therefore X = 1 \quad \therefore$ point of tangency is $(1,1)$

<u>Equation of the tangent</u> $\frac{Y - 1}{X - 1} = \frac{1}{2}$

Answer the following questions 20 questions **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $X = 2n^2 + 7$, $Y = \sqrt{n^3}$, n=1 , then $\frac{dY}{dX} =$

① $\frac{3}{8}$ ② $\frac{3}{4}$

③ 2 ④ 6

$\frac{dX}{dn} = 4n \quad \frac{dY}{dn} = \frac{3}{2}n^{\frac{1}{2}} \quad \therefore \frac{dY}{dX} = \frac{3}{2}n^{\frac{1}{2}} \times \frac{1}{4n}$ when n == 1 $\therefore \frac{dY}{dX} = \frac{3}{8}$

Q(2) Which of the following functions satisfies the relation $\frac{d^3Y}{dX^3} = Y$

① $Y = \frac{1}{12}(X+1)^4$ ② $Y = \sin X$

③ $Y = \frac{X}{X-1}$ ④ $Y = e^x$

$Y = e^X$

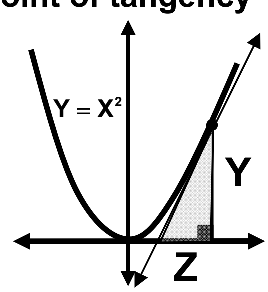
Q(3) If $F(X) = 2^X$ then $F'(3) =$……………

① 8 ln 3 ② 8 ln 2

③ 9 ln 2 ④ 9 ln 3

$F'(X) = \ln(2)2^X$ when X=3 $\therefore F'(X) = \ln(2)2^3 = 8\ln 2$

Q(4) A point is moving on the curve of $Y = X^2$ the rate change of the area of triangle formed from the tangent to the curve and the perpendicular from the point of tangency and the X-axis If $\frac{dX}{dt} = 4cm/sec$ and X=2cm is equals …..



① $4cm^2/sec$ ② $12cm^2/sec$

③ $8cm^2/sec$ ④ $10cm^2/sec$

$A = \frac{1}{2}Z \times Y \because \frac{dY}{dX} = 2X = \tan\theta \therefore \tan\theta = \frac{Y}{Z} = 2X \therefore Z = \frac{Y}{2X} \therefore$

$A = \frac{1}{2} \times \frac{Y}{2X} \times Y = \frac{1}{4} \times \frac{X^2}{X} \times X^2 = \frac{1}{4}X^3 \therefore \frac{dA}{dt} = \frac{3}{4}X^2 \frac{dX}{dt} = 12cm^2/sec$

Q(5) $\lim_{h\to 0}\dfrac{\tan\left(\frac{\pi}{3}+2h\right)-\sqrt{3}}{h}=$..........

① -8 ② 4

③ 8 ④ -4

$F^{/}(\tan 2X)=2\sec^2 2X$ when $X=\dfrac{\pi}{6}$ $2\left(\dfrac{1}{\cos 60^\circ}\right)^2=8$

Q(6) The ratio between the slopes of the two curves $Y=\ln 3\sqrt{X+1}$ and $Y=\ln 5\sqrt{X+1}$ when X=a is

① 3:5 ② 1:1

③ 5:3 ④ ln3:ln5

$Y_1=\ln 3+\frac{1}{2}\ln(X+1)$, $Y_2=\ln 5+\frac{1}{2}\ln(X+1)$ **1 : 1**

Q(7) $\int\dfrac{1}{X\ln X^3}dX=$....

① $3\ln|X|$ ② $3\ln\ln|X|$

③ $\frac{1}{3}\ln|X|$ ④ $\frac{1}{3}\ln\ln|X|$

$\int\dfrac{1}{X\ln X^3}dX=\frac{1}{3}\int\dfrac{1}{X\ln X}dX=\frac{1}{3}\int\dfrac{\frac{1}{X}}{\ln X}dX=\frac{1}{3}\ln\ln|X|$

Q(8) $\lim_{X\to\infty}\left(\dfrac{X+2}{X-1}\right)^{X+4}=$.......

① e^2 ② 1

③ e^3 ④ e^4

$\lim_{X\to\infty}\left(\dfrac{X+2}{X-1}\right)^{X+4}=\lim_{X\to\infty}\left(\dfrac{X-1+3}{X-1}\right)^{X+4}=\lim_{X\to\infty}\left(1+\dfrac{3}{X-1}\right)^{X-1+5}$

$=\lim_{X\to\infty}\left(1+\dfrac{3}{X-1}\right)^{X-1}\times\lim_{X\to\infty}\left(1+\dfrac{3}{X-1}\right)^{5}=e^3\times 1=e^3$

Q(9) If $\int [F(X)+2]\,dX = X^3 + aX^2 + 9$ and $F(1)=7$ then a=…..

① 3 ② 12

③ 6 ④ 9

$F(X)+2 = 3X^2 + 2aX \quad \therefore F(1)+2 = 3\times 1^2 + 2a \quad \therefore 7+2 = 3+2a \quad \therefore a = 3$

Q(10) If $Y = ae^{2X} + \sin(\ln X)$ where a is constant and if $\frac{dY}{dX} = e^3 + 1$ at X=1 then the value of a=…..

① e ② $\frac{1}{2}e$

③ $2e$ ④ $3e$

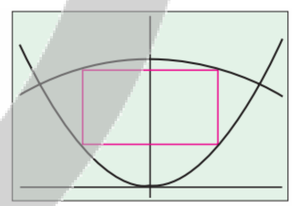
$\frac{dY}{dX} = 2ae^{2X} + \frac{1}{X}\cos(\ln X)$ when $X=1 \therefore \frac{dY}{dX} = 2ae^2 + \frac{1}{1}\cos\ln 1 = 2ae^2 + 1$

$\therefore e^3 = 2ae^2 \quad \therefore a = \frac{e}{2}$

Q(11) <u>In the given figure:</u> A rectangle is inscribed between $Y = 4X^2$ and $Y = 30 - X^2$ what is the maximum area of

① $20\sqrt{2}$ ② $40\sqrt{2}$

③ $30\sqrt{2}$ ④ 50



$A = 2X(30 - X^2 - 4X^2) = 2X(30 - 5X^2) = 60X - 10X^3$

$A' = 60 - 30X^2 \quad \therefore X = \sqrt{2} \quad \therefore A = 40\sqrt{2}$

Q(12) $\int_1^e \left(\frac{X^2-1}{X}\right) dX =$

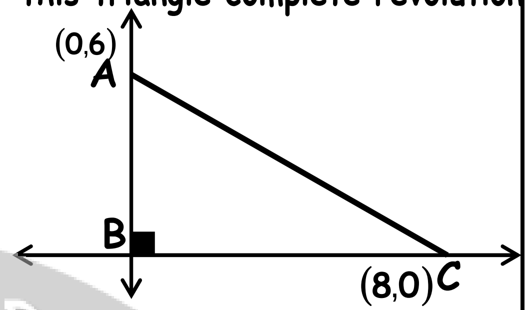
① $e - \frac{1}{e}$ ② $\frac{e^2}{2} - \frac{3}{2}$

③ $e^2 - e$ ④ $e^2 - 2$

$$\int_1^e X - \frac{1}{X}dX = \left[\frac{X^2}{2} - \ln X\right]_1^e = \left(\frac{e^2}{2} - \ln e\right) - \left(\frac{1^2}{2} - \ln 1\right)$$

$$= \left(\frac{e^2}{2} - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{e^2}{2} - \frac{3}{2}$$

Q(13) Right angled triangle the length of the side of the right angle 6cm and 8cm find the volume of the solid generated by revolving of this triangle complete revolution about the side of length 8cm



Slope of $\overleftrightarrow{AC} = \frac{6-0}{0-8} = -\frac{3}{4}$

Equation of $\overline{AC}$ $Y = -\frac{3}{4}X + 6$

Volume $= \pi\int_0^8 \left(-\frac{3X}{4} + 6\right)^2 dX = \pi\int_0^8 \left(\frac{9}{16}X^2 + 36 - 9X\right)dX =$

$= \pi\left[\frac{9X^3}{16 \times 3} + 36X - \frac{9X^2}{2}\right]_0^8 = 96\pi$

Q(14) Find the equation of tangent to the curve $Y = (X-1)e^X + 3\ln X + 2$ At the point (1,2)

$\frac{dY}{dX} = (X-1).e^x + e^x(1) + \frac{3}{X}$

$F'(X) = e^X + (X-1)e^X \quad \therefore F'(1) = e$

$\frac{Y-2}{X-1} = e$

Q(15) Determine The intervals of increasing and decreasing of the function $Y = \frac{1+X+X^2}{1-X+X^2}$, find its points of local maximum , local minimum

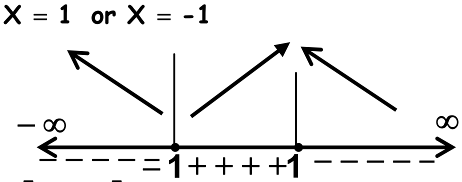
$\frac{dY}{dX} = \frac{(1+2X)(1-X+X^2) - (-1+2X)(1+X+X^2)}{(1-X+X^2)^2}$

$= \frac{1 - X + X^2 + 2X - 2X^2 + 2X^3 + 1 + X + X^2 - 2X - 2X^2 - 2X^3}{(1-X+X^2)^2} = \frac{-2X^2+2}{(1-X+X^2)^2} = \frac{-2(X^2-1)}{(1-X+X^2)^2}$

$X^2 - 1 = 0 \quad \therefore (X-1)(X+1) = 0 \quad \therefore X = 1$ or $X = -1$

The point $\left(-1, \frac{1}{3}\right)$ is local minimum

The point (1,3) is local maximum



Decrease $[-1,1]$ **increase** $]-1,1[$

Q(16) Find $\int (2X-3)\sqrt{X^2-3X+5}\,dX$

Let $Z = X^2 - 3X + 5 \quad \therefore dZ = (2X-3)dX \quad \therefore dX = \dfrac{dZ}{2X-3}$

$$\int (2X-3)\sqrt{X^2-3X+5}\,dX = \int (2X-3)\sqrt{Z}\,\frac{dZ}{2X-3} = \int \sqrt{Z}\,dZ = \frac{2}{3}Z^{\frac{3}{2}} + C$$

$$\frac{2}{3}(2X-3)^{\frac{3}{2}} + C$$

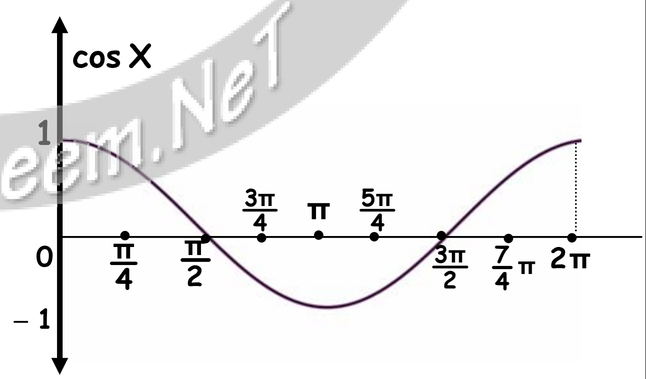
Q(17) Using one of the integration techniques to find $\int X(\sin X + \cos X)^2 dX$

$$= \int X(\sin^2 X + \cos^2 X + 2\sin X \cos X)dX$$
$$= \int X(1 + \sin 2X)dX$$

D	I
X	$1 + \sin 2X$
1	$X - \frac{1}{2}\cos 2X$

$$= X\left(X - \frac{1}{2}X\cos 2X\right) - \int\left(X - \frac{1}{2}\cos 2x\right)dX$$
$$= X\left(X - \frac{1}{2}X\cos 2X\right) - \frac{X^2}{2} - \frac{1}{4}\sin 2X$$
$$= \frac{X^2}{2} - \frac{1}{2}X^2\cos 2X - \frac{1}{4}\sin 2X$$

Q(18) Find the area between the two curves $y = \cos \pi X$ and X-axis In $[0,2]$



$\cos \pi X = 0 \;\therefore$

$\pi X = \dfrac{\pi}{2} \rightarrow X = \dfrac{1}{2}$

or $\pi X = \dfrac{3\pi}{2} \rightarrow X = \dfrac{3}{2}$

$$A = \int_0^{\frac{1}{2}} \cos \pi X dX - \int_{\frac{1}{2}}^{\frac{3}{2}} \cos \pi X dX + \int_{\frac{3}{2}}^{2} \cos \pi X dX$$

$$= \left[\frac{\sin \pi X}{\pi}\right]_0^{\frac{1}{2}} - \left[\frac{\sin \pi X}{\pi}\right]_{\frac{1}{2}}^{\frac{3}{2}} + \left[\frac{\sin \pi X}{\pi}\right]_{\frac{3}{2}}^{2} = \frac{4}{\pi}$$

Q(19) If the slope of the tangent equals $\frac{e^{-Y}(X+4)(X-3)}{X^2-3X}$ and the curve passing through the point (1,0) find the equation of the curve

$$\frac{dY}{dX}=\frac{e^{-Y}(X+4)(X-3)}{X^2-3X}=\frac{e^{-Y}(X+4)(X-3)}{X(X-3)}=\frac{e^{-Y}(X+4)}{X}$$

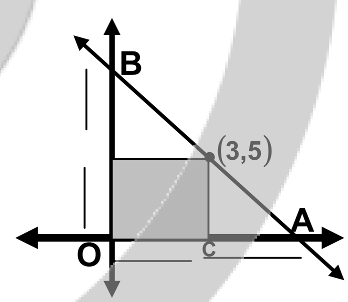
$$e^{Y}dY=\frac{X+4}{X}dX \quad \therefore \int e^{Y}dY=\int\frac{X+4}{X}dX$$

$$\int e^{Y}dY=\int 1+\frac{4}{X}dX \quad \therefore e^{Y}=X+4\ln X+C$$

$$\therefore e^{0}=1+4\ln 1+C \quad \therefore 1=1+0+C \quad \therefore C=0$$

$$\therefore e^{Y}=X+4\ln X$$

Q(20) In the opposite figure calculate the smallest area of triangular region in the first quadrant inscribed between the line passing the point (3,5) and the two axes



$OA=X$, $OB=Y$ $AC=X-3$

$$\tan BAO=\frac{Y}{X}=\frac{5}{X-3}\therefore Y=\frac{5X}{X-3}$$

$$\text{Area of } \Delta\Delta OA=\frac{1}{2}\times X\times Y=\frac{5}{2}\times\frac{X^2}{X-3}$$

$$\frac{dA}{dX}=\frac{5}{2}\times\frac{2X(X-3)-X^2}{(X-3)^2}=0\therefore X^2-6X=0 \therefore X=6$$

☆If the tangent to the curve $Y = X^2$ passes through the point (3,5) find the equation of the tangent

Let the point of tangency (X , Y)

$$\therefore \frac{Y-5}{X-3}=2X \quad \therefore \frac{X^2-5}{X-3}=2X \quad \therefore X^2-5=2X^2-6X$$

$$\therefore X^2-6X+5=0 \quad \therefore X=5 \quad \text{or } X=1$$

points of tangency are (5,25) or (1,1)

$$\text{Tangent equation is } \frac{Y-25}{X-5}=10 \quad \text{or } \frac{Y-1}{X-1}=2$$

Answer the following questions | 20 questions **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $Y = \frac{m-1}{m+1}$ and $X = \frac{m+1}{m-1}$ then $\frac{dY}{dX} = \ldots$ when X=1

① 0 ② 1

③ $\frac{1}{2}$ ④ -1

$$XY = \frac{m-1}{m+1} \times \frac{m+1}{m-1} = 1 \quad \therefore \quad XY = 1 \quad \therefore \quad Y = \frac{1}{X} = X^{-1} \quad \therefore \frac{dY}{dX} = -X^{-2} = -1(1)^{-2} = -1$$

Q(2) If $X^2 + Y^2 = 25$ then $\frac{d^2Y}{dX^2}$ at (3,4) equals............

① $\frac{25}{27}$ ② $-\frac{25}{64}$

③ $-\frac{25}{27}$ ④ $\frac{7}{27}$

$$2X + 2Y\frac{dY}{dX} = 0 \quad \therefore \frac{dY}{dX} = \frac{-X}{Y} \quad \therefore \frac{d^2Y}{dX^2} = \frac{-Y + X\frac{dY}{dX}}{Y^2} = \frac{-4 + 3 \times \frac{-3}{4}}{4^2} = -\frac{25}{64}$$

Q(3) If $\sin X = e^Y$ and $0 < X < \pi$ then $\frac{dY}{dX} =$..........

① tanX ② secX

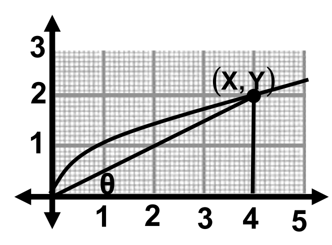
③ cotX ④ cscX

$$\cos X = e^Y \frac{dY}{dX} \quad \therefore \frac{dY}{dX} = \frac{\cos X}{e^Y} = \frac{\cos X}{\sin X} = \cot X$$

Q(4) a particle (X,Y) is moving along the curve of the function $Y = \sqrt{X}$ when X=4 , the Y –component of the position of the particle is increaseing at rate 1cm/sec the rate of change of the angle of inclination θ at the same moment ?

① 5 ② 4

③ $-\frac{1}{5}$ ④ 3



$\frac{dY}{dt} = \frac{1}{2}X^{-\frac{1}{2}}\frac{dX}{dt}$ when $X = 4$ $Y = 2, \frac{dX}{dt} = 4cm/sec$

$\tan\theta = \frac{Y}{X} = X^{-\frac{1}{2}} \quad \therefore \left(\sec^2\theta\right)\frac{d\theta}{dt} = -\frac{1}{2}X^{-\frac{3}{2}}\frac{dX}{dt}$

$X = 4 \quad , \frac{dX}{dt} = 4 , \sec\theta = \frac{2\sqrt{5}}{4} \quad \therefore \frac{d\theta}{dt} = \left(-\frac{1}{2}(4)^{-\frac{3}{2}} \times 4\right) \div \left(\frac{2\sqrt{5}}{4}\right)^2 = -\frac{1}{5} rad/min$

Q(5) The slope of the tangent of the graph of $Y = \ln(X^2)$ at $X=e^2$ =…..

① $\frac{1}{e^2}$ ② $\frac{2}{e^2}$

③ $\frac{3}{e^2}$ ④ $\frac{4}{e^2}$

$Y = 2\ln X \quad \therefore \frac{dY}{dX} = \frac{2}{X}$ when $X=e^2$ $\quad \therefore \frac{dY}{dX} = \frac{2}{e^2}$

Q(6) If $Y = e^{nX}$ then $\frac{d^nY}{dX^n}$ =…….

① $n^n e^{nX}$ ② $\lfloor n \, e^X$ =

③ $\lfloor n \, e^{nX}$ = ④ ne^{nX}

$n^n e^{nX}$

Q(7) $\lim\limits_{h\to 0} \frac{\ln(4+h)-\ln 4}{h}$

① 4 ② ln4

③ $\frac{1}{4}$ ④ e

$\lim\limits_{h\to 0} \frac{\ln(4+h)-\ln 4}{h} = F'(\ln X) = \frac{1}{X} = \frac{1}{4}$

Q(8) The absolute maximum value of the function $F(X) = \frac{\ln X}{X}$

① e ② Not exist

③ $\frac{1}{e}$ ④ 1

$F'(X) = \frac{\frac{1}{X} \times X - 1\ln X}{X^2} \quad \therefore \frac{1}{X} \times X - 1\ln X = 0 \quad \therefore 1 - \ln X = 0$

$\ln X = 1 \quad \therefore X = e \quad \therefore F(e) = \frac{\ln e}{e} = \frac{1}{e}$

Q(9) If $\int_{-2}^{2}\left(X^{7}+K\right)dX=16$ then K=…

① -12 ② -4

③ 12 ④ 4

$\left[\frac{X^{8}}{8}+KX\right]_{-2}^{2}=16 \therefore [32+2K]-32+2K=16 \quad \therefore 4K=16 \therefore K=4$

Q(10) $\int_{0}^{\frac{\pi}{3}}\sin(3X)dX=$……..

① -2 ② 2

③ $-\frac{2}{3}$ ④ $\frac{2}{3}$

$\left[-\frac{1}{3}\cos 3X\right]_{0}^{\frac{\pi}{3}}=-\frac{1}{3}(\cos\pi-\cos 0)=\frac{2}{3}$

Q(11) The area of the region bounded by the curve $Y=-X^{2}+X+6$ and the line $Y=4$

① $\frac{3}{2}$ ② $\frac{7}{3}$

③ $\frac{9}{2}$ ④ $\frac{33}{2}$

$-X^{2}+X+6=4 \therefore X^{2}-X-2=0 \therefore (X-2)(X+1)=$

$\int_{-1}^{2}\left(-X^{2}+X+6-4\right)dX=\left[-\frac{X^{3}}{3}+\frac{X^{2}}{2}+2X\right]_{-1}^{2}=\frac{9}{2}$

Q(12)The function F has a continuous derivative . the following table gives the values of F and its derivative for X=0 and X=4

If $\int_{0}^{4}f(X)dX=8$ the value of $\int_{0}^{4}Xf^{/}(X)dX=$………

X	F(X)	$F^{/}(X)$
0	2	5
4	-3	11

① -20 ② -13

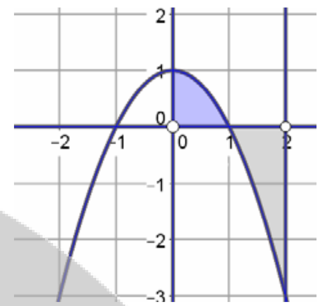
③ -12 ④ -7

$[XF(X)]_{0}^{4}-\int_{0}^{4}F(X)dX=4\times F(4)-0\times F(0)-8$

$=4\times -3-8=-20$

D	I
X	$F^{/}(X)$
1	F(X)

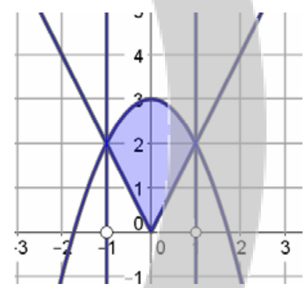
Q(13) Find the area bounded between curve $Y = 1 - X^2$ and X axis from $[0,2]$



$$\int_0^1 (1 - X^2)\,dX + \int_1^2 (X^2 - 1)\,dX$$

$$\left[X - \frac{X^3}{3}\right]_0^1 + \left[\frac{X^3}{3} - X\right]_1^2 = \frac{2}{3} + \frac{4}{3} = 2$$

Q(14) Find the area of the region bounded by the two curves $y = 3 - X^2$, $y = |2X|$



$3 - X^2 = 2X \quad \therefore X^2 + 2X - 3 = 0$

$(X + 3)(X - 1) = 0 \quad \therefore X = 1$

$3 - X^2 = -2X \quad \therefore X^2 - 2X - 3 = 0$

$(X - 3)(X + 1) = 0 \quad \therefore X = -1$

$$A = \int_0^1 \left[(3X^2) - (2X)\right] dX + \int_{-1}^0 \left[(3X^2) - (2X)\right] dX = \frac{10}{3}$$

Q(15) $\overline{AB}$ is a line segment itys length 8cm its end A move on the positive X-axis and its end B moves on the positive Y-axis such that $\angle ABO$ increase by rate $\frac{1}{20}$ rad/sec (o is the origin) find the rate of change of the area of triangle APB when A a distance 4cm from O

$A = \frac{1}{8} \times 8 \times Y \times \sin\theta = 4Y\sin\theta$

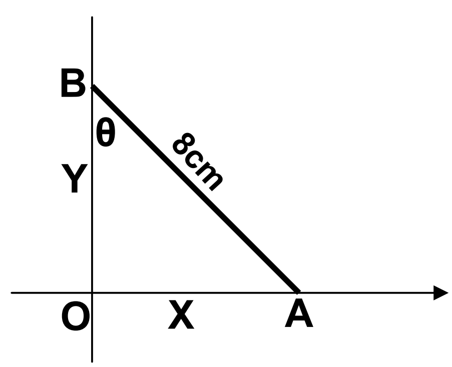
$\cos\theta = \frac{Y}{8} \quad \therefore Y = 8\cos\theta$

$\therefore A = 4 \times 8 \times \cos\theta \times \sin\theta = 16 \times 2\cos\theta\sin\theta$

$A = 16\sin 2\theta \quad \therefore \frac{dA}{dt} = 16 \times 2\cos 2\theta \times \frac{d\theta}{dt}$

when $X = 4 \quad \therefore \sin\theta = \frac{1}{2} \quad \therefore \theta = 30°$

$\therefore \frac{dA}{dt} = 32 \times \frac{1}{2} \times \frac{1}{20} = 0.8$



Q(16) Using one of the integration techniques to find

$$\int_0^3 X\sqrt{X^2+3}\,dX$$

Let $Z = X^2 + 3 \quad \therefore \frac{dZ}{dX} = 2X \quad \therefore \ dX = \frac{dZ}{2X}$

$$\int_0^3 XZ^{\frac{1}{2}}\,\frac{dZ}{2X} = \frac{1}{2}\int_0^3 Z^{\frac{1}{2}}\,dZ = \frac{1}{2}\times\frac{2}{3}\left[Z^{\frac{3}{2}}\right] = \frac{1}{3}\left[\left(X^2+3\right)^{\frac{3}{2}}\right]_0^3 =$$

Q(17) Using one of the integration techniques to find

$$\int_{-\frac{\pi}{3}}^{0} \sin^2 X\cos^2 X\,dX$$

$$\int_{-\frac{\pi}{3}}^{0}\left(\frac{1}{2}\sin 2X\right)^2 dX = \frac{1}{4}\int_{-\frac{\pi}{3}}^{0}(\sin 2X)^2\,dX$$

$$\frac{1}{4}\int_{-\frac{\pi}{3}}^{0}\left(\frac{1}{2}-\frac{1}{2}\sin 4X\right)dX = \frac{1}{4}\left[\frac{1}{2}X+\frac{1}{8}\cos 4X\right]_{-\frac{\pi}{3}}^{0}$$

Q(18) If $\int\left[F'(X)+X^2\right]dX = 2X^3 + CX^2 + 2$ and $F'(1) = 4$ and $F(2) = 6$ find $F(-1)$

$\therefore F'(X) + X^2 = 6X^2 + 2CX \quad \therefore 4 + 1 = 6 + 2C \quad \therefore C = -\frac{1}{2}$

$F'(X) = 5X^2 - X \quad \therefore F(X) = \int\left(5X^2 - X\right)dX = \frac{5}{3}X^3 - \frac{1}{2}X^2 + C$

$\because F(2) = 6 \quad \therefore C = -\frac{16}{3} \quad \therefore F(X) = \frac{5}{3}X^3 - \frac{1}{2}X^2 - \frac{16}{3}$

Q(19) If the slope of the tangent equals 2XY and the curve passing through the point (2,1) find value of Y when X=3

$\frac{dY}{dX} = 2XY \quad \therefore \quad \int \frac{dY}{Y} = \int 2XdX \quad \therefore \quad \ln Y = X^2 + C$

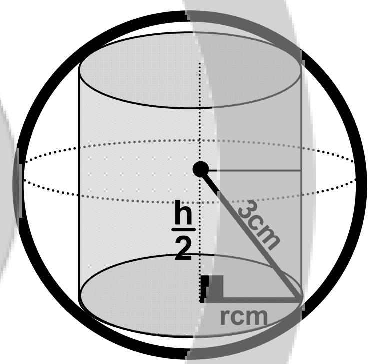
$\ln 1 = 2^2 + C \quad \therefore 0 = 4 + C \quad \therefore C = -4$

$\therefore \ln Y = X^2 + C$ when $X = 3 \quad \therefore \ln Y = 9 - 4 = 5$

$\therefore \ln Y = 5 \quad \therefore Y = e^5$

Q(20) Find the dimensions of the cylinder of maximum volume which can be inscribed in a sphere of radius 3cm

Let r is the radius of the cylinder h its height



$r^2 + \left(\frac{h}{2}\right)^2 = 3^2 \quad \therefore r^2 + \frac{h^2}{4} = 9 \quad \therefore r^2 = 9 - \frac{h^2}{4}$

Volume of the cylinder

$V = \pi r^2 \times h = \pi \times h \times \left(9 - \frac{h^2}{4}\right) = \pi\left(9h - \frac{h^3}{4}\right)$

$\frac{dV}{dX} = \pi\left(9 - \frac{3}{4}h^2\right) = 0 \quad \therefore \frac{3}{4}h^2 = 9 \quad \therefore h^2 = \frac{3}{4} \quad \therefore h = \frac{\sqrt{3}}{2}$ cm

$\therefore r^2 = 9 - \left(\frac{\sqrt{3}}{2}\right)^2 = 9 - \frac{9}{4} = \frac{27}{4} \quad \therefore r = \frac{3\sqrt{3}}{2}$ cm

$\therefore V =$

$V = \pi\left(\frac{3\sqrt{3}}{2}\right)^2 \times \frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{8}\pi \quad cm^3$

Find the equation of the tangents to the curve $XY = 8$ at which are parallel to the line $Y + 2X = 9$

$Y + X\frac{dY}{dX} = 0 \quad \therefore \frac{dY}{dX} = -\frac{Y}{X}$ the line its slope $-2 \quad \therefore -\frac{Y}{X} = -2 \quad \therefore Y = 2X$

$\because XY = 8 \quad \therefore X(2X) = 8 \quad \therefore 2X^2 = 8 \quad \therefore X^2 = 4 \quad \therefore X = \pm 2 \quad \therefore Y = \pm 4$

the two points are $(2,4)$, $(-2,-4)$

<u>Equation of the first tangent</u> $\frac{Y-4}{X-2} = -2 \quad \therefore Y - 4 = -2X + 4 \quad \therefore Y + 2X - 8 = 0$

<u>Equation of the second tangent</u> $\frac{Y+4}{X+2} = -2 \quad \therefore Y + 4 = -2X - 4 \quad \therefore Y + 2X + 8 = 0$

Answer the following questions | 20 questions | **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $F(X) = X^n \sin X$ then $F'(X) =$ ....

① $X^{n-1}(X\cos X + n\sin X)$ ② $X^n(X\cos X + n\sin X)$

③ $X^{n-1}(X\cos X + \sin X)$ ④ $X^{n-1}(\cos X + \sin X)$

$F'(X) = nX^{n-1}\sin X + X^n\cos X = X^{n-1}(n\sin X + X\cos X)$

Q(2) If X and Y two positive integers such that 3X+Y=60 Then the greatest possible value of XY is

① 75 ② 300

③ 50 ④ 200

$Y = 60 - 3X \quad \therefore F(X) = XY = X(60-3X) = 60X - 3X^2 \quad \therefore F'(X) = 60 - 6X$

$\therefore 60 - 6X = 0 \quad \therefore X = 10 \quad \therefore Y = 30 \quad \therefore XY = 300$

Q(3) A point moves on the curve $XY^2 = 12$ At point $(3,2)$ then $\frac{dY}{dX} =$

① -4 ② 3

③ $-\frac{1}{3}$ ④ $-\frac{3}{2}$

$Y^2 + 2XY\frac{dY}{dX} = 0 \quad \therefore 4 + 12\frac{dY}{dX} = 0 \quad \therefore \frac{dY}{dX} = -\frac{4}{12} = -\frac{1}{3}$

Q(4) If the radius length of a circle increase at a rate $\frac{4}{\pi}$ cm/sec Then the circumference increase at this moment at the rate ….cm/sec

① $\frac{4}{\pi}$ ② $\frac{\pi}{4}$

③ $\frac{1}{8}$ ④ 8

$C = 2\pi r \quad \therefore \frac{dC}{dt} = 2\pi\frac{dr}{dt} = 2\pi \times \frac{4}{\pi} = 8$

Q(5) If $\sin(X - Y) = X - Y$ then $\frac{dY}{dX} = \ldots$

① $\cos(X - Y)$ ② $1 - \cos(X - Y)$

③ $\text{Sin}(X - Y)$ ④ 1

$\left(1 - \frac{dY}{dX}\right)\cos(X - Y) = 1 - \frac{dY}{dX} \quad \therefore \cos(X - Y) - \frac{dY}{dX}\cos(X - Y) = 1 - \frac{dY}{dX}$

$\frac{dY}{dX}\cos(X - Y) - \frac{dY}{dX} = \cos(X - Y) - 1 \quad \therefore \frac{dY}{dX} = 1$

Q(6) If $F(X) = e^{3\ln X}$ then $F'(1) = \ldots$

① 2 ② 1

③ e ④ 3

$F(X) = e^{\ln X^3} = X^3 \rightarrow F'(X) = 3X^2 \Rightarrow F'(1) = 3(1)^2 = 3$

Q(7) The derivative of the area of a square with respect to its diagonal (Z) when $Z = \sqrt{5}$ is …

① $\frac{1}{2}$ ② $\sqrt{5}$

③ $\sqrt{3}$ ④ $\frac{1}{\sqrt{2}}$

$A = \frac{1}{2}Z^2 \quad \therefore \frac{dA}{dZ} = Z = \sqrt{5}$

Q(8) $\frac{d}{dX}Xe^{\ln X^2} = \ldots$

① $1+2X$ ② $X + X^2$

③ $3X^2$ ④ X^3

$Xe^{\ln X^2} = X^3 \quad \therefore \frac{d}{dX}X^3 = 3X^2$

Q(9) $\int_1^{10} \log_{X^3} X^5 =$ …………..

① 8 ② -8

③ 15 ④ -15

$\int_1^{10} \frac{\ln X^5}{\ln X^3} dX = \int_1^{10} \frac{5 \ln X}{3 \ln X} dX = \int_1^{10} \frac{5}{3} dX = \left[\frac{5}{3} X\right]_1^{10} = 15$ **remark** $\log_Y X = \frac{\ln X}{\ln Y}$

Q(10) $\int_{-1}^{1} \left(X^4 + \sin^{11} X\right) dX =$ ………………

① $\frac{1}{5}$ ② 0

③ $\frac{2}{5}$ ④ 2

$\int_{-1}^{1} X^4 dX + \int_{-1}^{1} \sin^{11} X dX = \left[\frac{X^5}{5}\right]_{-1}^{1} + \text{zero} = \frac{2}{5}$

$\int_{-1}^{1} \sin^{11} X dX$ =because it is odd function

Q(11) $\int_0^1 5^X \ln 5 \, dX =$ ………

① 4 ② 20

③ 5 ④ 3

$\ln 5 \int_0^1 5^X dX = \ln 5 \times \frac{5^X}{\ln 5} = \left[5^X\right]_0^1 = 5 - 1 = 4$

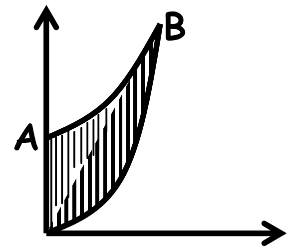
Q(12) $\int_1^2 \frac{\lg_3 X}{\ln X} dX =$ ………

① $\frac{2}{\ln 3}$ ② ln3

③ $\frac{\ln 3}{2}$ ④ $\frac{1}{\ln 3}$

$\int_1^2 \frac{\frac{\ln X}{\ln 3}}{\ln X} dX = \int_1^2 \frac{1}{\ln 3} dX = \left[\frac{1}{\ln 3} X\right]_1^2 = \frac{1}{\ln 3}(2 - 1) = \frac{1}{\ln 3}$

Q(13) The opposite figure shows the graphs of the functions $F(X) = X^3 + 8$ and $g(X) = X^3 + 2X$ (i)find the two points A and B
(ii) find the shaded area
(iii) find The line X=C which divides the shaded area into two equals part



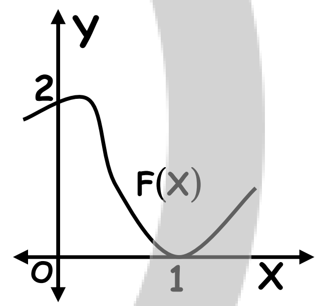
$$\int_0^C \left[\left(X^3 - 8\right) - \left(X^3 + 2X\right)\right] dX$$

$$= \int_0^C [8 - 2X] dX = \left[8X - X^2\right]_0^C = 8C - C^2$$

$$\therefore 8 = 8C - C^2 \quad \therefore C^2 - 8C + 8 = 0$$

$$C = 4 - \sqrt{8}$$

Q(14) In the given figure : find $\int_0^1 [\,F(X)\,]^2 \, F'(X)\, dX$



$$\int_0^1 [\,F(X)\,]^2 F'(X)\, dX = \frac{1}{3}\left[[F(X)]^3\right]_0^1$$

$$= \frac{1}{3}\left[F(1)^3 - F(0)^3\right] = \frac{1}{3}\left[0^3 - 2^8\right] = -\frac{256}{3}$$

Q(15) If $F(X) = X^2 - \dfrac{8}{X}$ **discuss the convexity if found and find the inflection point**

$$F'(X) = 2X + \frac{8}{X^2}$$

$$F''(X) = 2 - \frac{16}{X^3} = \frac{2X^3 - 16}{X^3}$$

$$\therefore 2X^3 - 16 = 0$$

$$\underbrace{X = 0}_{\text{refused}} \text{ or } X = 2$$

Q(16) Find $\int e^{\sqrt{X}} dX$

let $\sqrt{X} = U \quad \therefore \frac{dU}{dX} = \frac{1}{2\sqrt{X}} \quad \therefore dX = 2\sqrt{X}dU = 2UdU$

$2\int e^{U} U dU$

D	I
U	e^{U}
1	e^{U}

$\therefore 2\int e^{U} U dU = 2Ue^{U} - e^{U} = 2\sqrt{X} = e^{\sqrt{X}}$

Q(17) Using one of the integration techniques to find $\int_0^{\frac{\pi}{2}} \sqrt{1 + \sin X}\, dX$

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin^2 \frac{X}{2} + \cos^2 X \frac{X}{2} + 2\sin \frac{X}{2} \cos \frac{X}{2}}\, dX$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\left(\sin \frac{X}{2} + \cos \frac{X}{2}\right)^2}\, dX = \int_0^{\frac{\pi}{2}} \left(\sin \frac{X}{2} + \cos \frac{X}{2}\right) dX =$$

$$\left[-2\cos \frac{X}{2} + 2\sin \frac{X}{2}\right]_0^{\frac{\pi}{2}} = 2$$

Q(18) Find the equation of the tangent and normal to the curve $2 + \ln X \ln Y = X^2 + Y$ at the point which X-coordinate is 1

$\frac{1}{X}\ln Y + \frac{Y}{Y'}\ln X = 2X + Y'$ at X=1

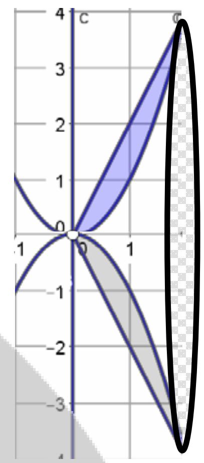
When X=1 $\quad \therefore 2 + \ln 1 \ln Y = 1^2 + Y \quad \therefore 2 = 1 + Y \quad \therefore Y = 1$

$\frac{1}{1}\ln 1 + \frac{1}{Y'}\ln 1 = 2 \times 1 + Y' \quad \therefore 0 = 2 + Y' \quad \therefore Y' = -2$

Equation of tangent : $\frac{Y-1}{X-1} = -2$

Equation of normal : $\frac{Y-1}{X-1} = \frac{1}{2}$

Q(19) Find the volume of the solid generated by revolving the region bounded by the curve $y = X^2$ and the straight line $y = 2X$ a complete revolution about x-axis



$X^2 = 2X \quad \therefore X^2 - 2X = 0 \quad \therefore X(X - 2) = 0$

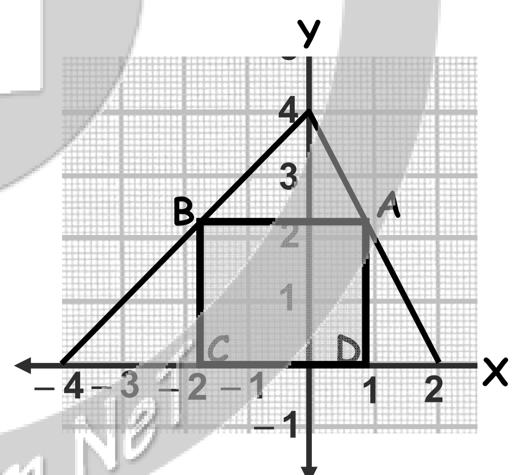
$X = 0$ or $X = 2$

$V = \int_0^2 \left(2X - X^2\right)^2 dX$

$V = \int_0^2 \left(2X - X^2\right)^2 dX$

$V = \int_0^2 \left(4X^2 + X^4 - 4X^3\right)^2 dX = \left[\frac{4X^3}{3} + \frac{X^5}{5} - X^4\right]_0^2 = \frac{16}{15}$

Q(20) In the opposite figure A rectangle ABCD having its vertices C and D on the X-axes A lies on the line Y=4-2X and B lies on the line Y-4=X Find the greatest area of ABCD



$A = y\left(\frac{4 - Y}{2} - (Y - 4)\right)$

$A = Y\left(2 - \frac{1}{2}Y - Y + 4\right) = Y\left(-\frac{3}{2}Y + 6\right)$

$A = -\frac{3}{2}Y^2 + 6Y$

$\frac{dA}{dY} = -3Y + 6 = 0 \quad \therefore Y = 2$

$A = 6$

Find the point on the curve $X^2 + XY + Y^2 = 3$ at which the tangent is parallel to the Y axis

$2X + X\frac{dY}{dX} + Y \times 1 + 2Y\frac{dY}{dX} = 0 \quad \therefore \quad \frac{dY}{dX}(X + 2Y) = -(2X + Y)$

$\because$ the tangent is// Y – axes $\therefore \frac{dY}{dX} = -\frac{2X + Y}{X + 2Y} = \frac{1}{0} \quad \therefore X + 2Y = 0 \therefore X = -2Y$

$\therefore (-2Y)^2 - 2Y \times Y + Y^2 = 3 \quad \therefore 4Y^2 - 2Y^2 + Y^2 - 3 = 0 \quad \therefore 3Y^2 = 3 \quad \therefore Y^2 = 1$

$\therefore Y = \pm 1$ points are $(-2,1),(2,-1)$

Answer the following questions | 20 questions **Model Answer**

From 1 to 12 choose the correct answer

Q(1) If F(X)=cotX then $F''\left(\frac{\pi}{4}\right)$ equals :

① $-\frac{4}{9}$ ② $\frac{4}{9}$

③ $\frac{9}{2}$ ④ 4

$F'(X) = -\csc^2 X \quad \therefore F''(X) = 2\csc^2 X \cot X \quad = 2(2)(1) = 4$

Q(2) If $Y = \frac{3X-5}{X-2}$,X=1 then $\frac{d^3Y}{dX^3} =$

① -12 ② 6

③ 12 ④ -6

$\frac{dY}{dX} = \frac{3(X-2)-3X+5}{(X-2)^2} = -(X-2)^{-2} \therefore \frac{d^2Y}{dX^2} = 2(X-2)^{-3} \therefore \frac{d^3Y}{dX^3} = -6(X-2)^{-3} = -6$

Q(3) The slope of tangent to the line $Y = 2^{1-X}$ at X=2

① $-\frac{1}{2}$ ② $\frac{1}{2}\ln 2$

③ -2 ④ $\frac{-\ln 2}{2}$

$\frac{dY}{dX} = -\ln 2 \times 2^{1-X} = -\frac{1}{2}\ln 2$

Q(4) If the volume of a cube is increasing at 24 cm^3/ min and the surface area of the cube is increasing at 12 cm^2/ min, what is the length of each edge of the cube

① 2 ② 8

③ $2\sqrt{2}$ ④ 4

$V = X^3 \quad \therefore \frac{dV}{dt} = 3X^2\frac{dX}{dt} \quad \therefore 24 = 3X^2\frac{dX}{dt} \to (1)$

$A = 6X^2 \quad \therefore \frac{dA}{dt} = 12X\frac{dX}{dt} \quad \therefore 12 = 12X\frac{dX}{dt} \to (2) \quad \therefore$ by dividing

Q(5) If $F(X) = 2X^2 + X - 1$ and $g(X) = \sqrt{X}$ then $(F \circ g)'\left(\frac{1}{4}\right) =$

① -3 ② 3

③ $\frac{1}{2}$ ④ $\frac{-1}{2}$

$(F \circ g) = 2X + \sqrt{X} - 1 \quad \therefore (F \circ g)' = 2 + \frac{1}{2\sqrt{X}} = 3$

Q(6) If $\int_0^{\frac{\pi}{2}} 5^{\cos X} \sin X dX =$ ……………

① 4 ② $\frac{-4}{\ln 5}$

③ $\frac{4}{\ln 5}$ ④ 1

$\left[\frac{1}{\ln 5} 5^{\cos X}\right]_0^{\frac{\pi}{2}} = \left[\frac{1}{\ln 5} 5^0 - \frac{1}{\ln 5} 5^1\right] = -\frac{4}{\ln 5}$

Q(7) $\int_e^{2e} \frac{1}{X} dX =$

① e ② $\ln 2$

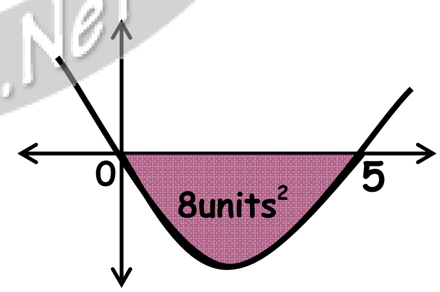
③ 2 ④ e^2

$[\ln X]_2^{2e} = \ln 2e - \ln e = \ln 2$

Q(8) In the given figure if the area bounded between the curcveof the functionF(X) and the X-axsis equal 8 units^2 then $\int_0^5 (1 - F(X)) dX =$ ..

① -3 ② 2

③ 13 ④ 3



$\int_0^5 (1 - F(X)) dX = \int_0^5 1 dX - \int_0^5 F(X) dX = [X]_0^5 + 8 = 5 + 8 = 13$

Q(9) If $\int_3^7 F(X)dX = 12$, then $\int_1^2 F(4X-1)dX = ...$

① 12 ② 4

③ 3 ④ 110

$at X = 1 \therefore Y = 4X - 1 \therefore Y = 3$ and $at X = 2 \quad Y = 4(2) - 1 = 7$

$let Y = 4X - 1 \therefore dY = 4dX \therefore \int_3^7 F(Y)dY = 3$

$\int_3^7 F(Y)dY = \frac{1}{4}\int_3^7 F(Y)dY = \frac{1}{4} \times 12 = 3$

Q(10) If $\frac{dY}{dX} = Y\cos X$ then Y= .... where a is constant

① $ae^{-\sin X}$ ② $ae^{\sin X}$

③ $ae^{\cos X}$ ④ $ae^{-\cos X}$

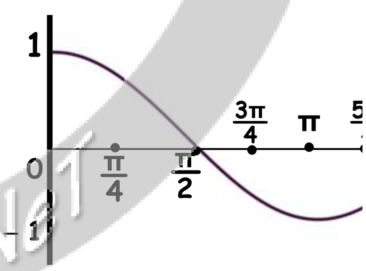
$\frac{dY}{Y} = \cos X dX \quad \therefore \int \frac{dY}{Y} = \int \cos X dX \quad \therefore \ln Y = \sin X + C$

$Y = e^{\sin X + C} = e^{\sin X} \times e^{C} = ae^{\sin X}$

Q(11) $\int_0^{\pi} |\cos X| \, dX$

① 1 ② 3

③ 2 ④ π



$\int_0^{\frac{\pi}{2}} \cos X \, dX - \int_{\frac{\pi}{2}}^{\pi} \cos X \, dX = [\sin X]_0^{\frac{\pi}{2}} - [\sin X]_{\frac{\pi}{2}}^{\pi}$

$= (1 - 0) - (0 - 1) = 2$

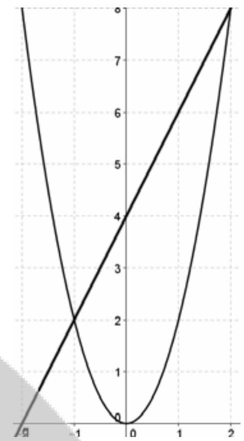
Q(12) If $\int_0^K (3X^2 - 1)dX = K^3 - 2$ then K=.....

① 0 ② 2

③ 1 ④ 3

$[X^3 - X]_0^k = K^3 - K = K^3 - 2 \therefore X = 2$

Q(13) Find the area bounded between the $Y = 2X^2$ and $Y=2X+4$



$2X^2 = 2X + 4 \quad \therefore \quad 2X^2 - 2X - 4 = 0$

$\therefore X^2 - X - 2 = 0 \quad \therefore X = -1 \text{ or } X = 2$

$A = \int_{-1}^{2} \left[(2X + 4) - 2X^2\right] dX$

$A = \left[X + 4X - \frac{2X^3}{3}\right]_{-1}^{2} \quad \therefore A = 9$

Q(14) If $Y = \tan^2 X$ prove that : $\frac{d^2Y}{dX^2} = 2(1 + Y)(1 + 3Y)$

$\frac{dY}{dX} = 2\tan X(\sec^2 X) = 2\tan X(1 + \tan^2 X) = 2(\tan X + \tan^3 X)$

$\therefore \frac{d^2Y}{dX^2} = 2(\sec^2 X + 3\tan^2 X \times \sec^2 X)$

$\therefore \frac{d^2Y}{dX^2} = 2(1 + \tan^2 X + 3\tan^2 X \times (1 + \tan^2 X))$

$\therefore \frac{d^2Y}{dX^2} = 2(1 + Y + 3Y \times (1 + Y)) = 2(1 + y)(1 + 3Y)$

Q(15) let $F(X) = \ln\left(\frac{X - 2}{X + 1}\right)$

find intervals of increase and decrease and discuss the convexity

$F(X) = \ln(X - 2) - \ln(X + 1)$

$F'(X) = \frac{1}{X - 2} - \frac{1}{X + 1} = \frac{X + 1 - X + 2}{(X - 2)(X + 1)} = \frac{3}{X^2 - X - 2}$

$\therefore X = 2$ or $X = -1$ oncrease $]-\infty, -1[\;,\;]2, \infty[$

$F''(X) = \frac{-3(2X - 1)}{(X^2 - X - 2)^2} = 0 \quad \therefore X = \frac{1}{2}$

Q(16) Using on of the integration techniques to find $\int X^2 \ln X dX$

D	I
$\ln X$	X^2
$\frac{1}{X}$	$\frac{X^3}{3}$

$$\int X^2 \ln X dX = \frac{1}{3}X^3 \ln X - \int \frac{1}{3}X^2 dX$$

$$= \frac{1}{3}X^3 \ln X - \frac{1}{9}X^3 + C$$

Q(17) Using one of the integration techniques to find

$$\int (\cos X + \sec X)^2 dX$$

$$\int \left(\cos^2 X + \sec^2 X + 2\right) dX =$$

$$\int \left(\frac{1}{2}(1 + \cos 2X) + \sec^2 X + 2\right) dX$$

$$= \frac{1}{2}\left(X + \frac{\sin 2X}{2}\right) + \tan X + 2x + C$$

Q(18) If $\frac{1}{2}\sqrt{Y}\frac{dY}{dX} + 6X - 1 = 0$ find the relation between X and Y

$$\frac{1}{2}\sqrt{Y}\frac{dY}{dX} = 1 - 6X$$

$$\int \frac{1}{2}\sqrt{Y} dY = \int (1 - 6X) dX$$

$$\int \frac{1}{2}Y^{\frac{1}{2}} dY = \int (1 - 6X) dX \qquad \therefore \frac{\frac{1}{2}}{\frac{3}{2}} Y^{\frac{3}{2}} = X - 3X^2 + C$$

Q(19) Find the equation of the tangent and normal line for
$\tan(XY) = X$ **at the point** $\left(1, \frac{\pi}{4}\right)$

$$\left[XY' + Y\right]\sec^2(XY) = 1 \quad \text{at } \left(1, \frac{\pi}{4}\right) \quad \therefore Y' = \frac{1}{2} - \frac{\pi}{4}$$

$$\left[Y' + \frac{\pi}{4}\right]\sec^2\frac{\pi}{4} = 1$$

$$Y' + \frac{\pi}{4} = \frac{1}{2} \quad \therefore Y' = \frac{1}{2} - \frac{\pi}{4}$$

equation of tangent :

$$\frac{Y - \frac{\pi}{4}}{X - 1} = \frac{1}{2} - \frac{\pi}{4}$$

Q(20) The height of a cone is 8m and radius of its base is 6cm find the dimensions of a cylinder with maximum volume that could be constructed inside the cone

$$\frac{8}{6} = \frac{8 - h}{r} \quad \therefore 8r = 48 - 6h$$

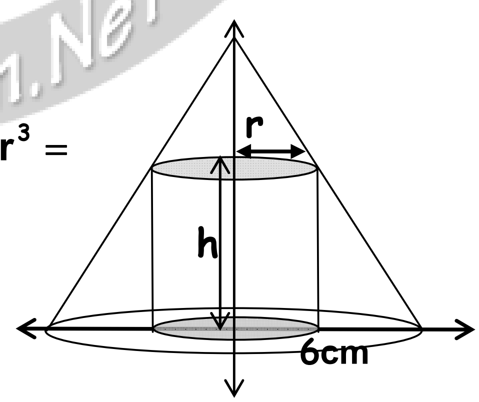
$$\therefore 6h = 48 - 8r \quad \therefore h = 8 - \frac{4}{3}r$$

$$\because V = \pi r^2 h \quad \therefore V(r) = \pi r^2\left(8 - \frac{4}{3}r\right) = 8\pi r^2 - \frac{4}{3}\pi r^3 =$$

$$V'(r) = 16\pi r - 4\pi r^2 = 4\pi r(4 - r) = 0 \quad \therefore r = 4$$

$$V'' = 16\pi - 8\pi r \qquad V''(4) < 0$$

$$h = 8 - \frac{4}{3}r = 8 - \frac{4}{3} \times 4 = \frac{8}{3}$$



Answer the following questions | 20 questions **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $Y = \cos 4X$ then $\frac{d^2X}{dX^2} = \ldots\ldots\ldots$ when $X=\frac{\pi}{2}$

① 0 ② -8

③ 16 ④ -16

$\frac{dY}{dX} = -4\sin 4X \quad \therefore \frac{d^2Y}{dX^2} = -16\cos 4X$ at $X = \frac{\pi}{2}$ $\quad \frac{d^2Y}{dX^2} = -16\cos 4\times\frac{\pi}{2} = -16$

Q(2) If $F(X) = \frac{e^X + 1}{e^X}$ then $F'(0) = \ldots\ldots\ldots$

① 1 ② -1

③ 0 ④ e

$F(X) = 1 + e^{-X} \quad \therefore F'(X) = -e^{-X} \quad \therefore F'(0) = -1$

Q(3) If $F(X) = (1+\sin X)^2$ then $F'\left(\frac{\pi}{2}\right) = \ldots\ldots\ldots$

① 0 ② 3

③ 4 ④ 12

$F'(X) = 2(1+\sin X)\cos X \quad = 2(1+\sin 90^\circ)\cos 90^\circ = \text{zeo}$

Q(4) A particle moves along the curve XY=10 if X=2 and $\frac{dY}{dt} = 3$ then $\frac{dX}{dt} = \ldots.$

① $-\frac{5}{2}$ ② $\frac{4}{5}$

③ $-\frac{6}{5}$ ④ $\frac{6}{5}$

$X\frac{dY}{dt} + Y\frac{dX}{dt} = 0 \quad \therefore 2\times 3 + 5\frac{dX}{dt} = 0 \quad \therefore \frac{dX}{dt} = -\frac{6}{5}$

Q(5) If $\ln(XY) = X$ then the slope of the tangent at X=1

① 0 ② e^2

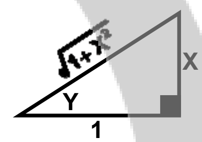
③ e ④ 1

$\frac{Y + XY'}{XY} = 1$ when $X = 1$ $\ln Y = 1$ $\therefore Y = e$ $\therefore \frac{e + Y'}{e} = 1$ $\therefore Y' = 0$

Q(6) If $X = \tan Y$ then $\frac{dY}{dX} =$

① $\frac{1}{1 + X^2}$ ② $\frac{1}{1 - X^2}$

③ $\frac{1}{\sqrt{1 + X^2}}$ ④ $\frac{X}{\sqrt{1 + X^2}}$



$1 = Y' \sec^2 Y$ $\therefore Y' = \frac{1}{\sec^2 Y} = \cos^2 Y = \frac{1}{1 + X^2}$

Q(7) If $F(X) = e^{\tan X}$ then $\lim_{X \to \frac{\pi}{4}} \frac{F(X) - F\left(\frac{\pi}{4}\right)}{X - \frac{\pi}{4}}$

① e ② e^2

③ 2e ④ $2e^2$

$F'(X) = e^{\tan X} \times \sec^2 X$ $\therefore F'\left(\frac{\pi}{4}\right) = e^{\tan 45^\circ} \times \sec^2 45^\circ = e \times \left(\frac{2}{\sqrt{2}}\right)^2 = 2e$

Q(8) If $F(X) = e^{\ln X^3 - 2X + 1}$ then $F'(0) =$ ……………

① -4 ① -1

③ 0 ③ -2

$F'(X) = \frac{3X^2 - 2}{X^3 - 2X + 1} e^{\ln X^3 - 2X + 1}$ $\therefore F'(0) = \frac{-2}{1} e^{\ln 1} = -2$

Q(9) If $\int (\tan^2 X - \sec^2 X) F(X) dX = 3 - X^2$ then F(X)=….

① 2X ② -2X

③ X^2 ④ 0

$\frac{d}{dX}\int -F(X) = \frac{d}{dX}(3 - X^2) \therefore -F(X) = -2X \quad \therefore F(X) = 2X$

Q(10) $\int_0^1 5^X \ln 5 dX = \ldots\ldots\ldots$

① 5 ② 3

③ 4 ④ 20

$[5^x]_0^1 = 5 - 1 = 4$

Q(11) If $\int_1^3 [2F(X) - 4] dX = 6$ and $\int_4^3 F(X) dX = -1$ then $\int_1^4 F(X) dX = \ldots\ldots\ldots$

① 7 ② 8

③ 10 ④ 5

$2\int_1^3 F(X) dX - [4X]_1^3 = 6 \therefore 2\int_1^3 F(X) dX - (12 - 4) = 6 \therefore 2\int_1^3 F(X) dX = 14$

$\therefore \int_1^3 F(X) dX = 7 \quad \therefore \int_1^4 F(X) dX = \int_1^3 F(X) dX + \int_3^4 F(X) dX = 7 + 1 = 8$

Q(12) If $Y = e^{\tan X} + a \ln \cos X + \int_0^{\frac{\pi}{3}} \frac{dX}{1 + \tan^2 X}$ and $\frac{dY}{dX} = 2e + 1$ when $X = \frac{\pi}{4}$ then a=…

① e ② 1

③ 2 ④ -1

$\frac{dY}{dX} = \sec^2 X e^{\tan X} - \frac{a \sin X}{\cos X} = \sec^2 X e^{\tan X} - a \tan X$

$\text{at} X = \frac{\pi}{4} \quad \therefore \frac{dY}{dX} = e(2) - a = 2e + 1 \quad \therefore a = -1$

Q(13) Find the area bounded between the two functions $XY = 2$ and $X + Y = 3$

$Y = \frac{2}{X}$ and $Y = 3 - X$ $\quad \frac{2}{X} = 3 - X \;\therefore X^2 - 3X + 2 = 0 \;\therefore X = 1$ or $X = 2$

$$\int_1^2 \left(3 - X - \frac{2}{X}\right)dX = \left[3X - \frac{X^2}{2} - \ln X\right]_1^2$$

$$= [6 - 2 - 2\ln 2] - \left[3 - \frac{1}{2} - 2\ln 1\right] = 4 - 2\ln 2 - \frac{5}{2} = \frac{3}{2} - 2\ln 2$$

Q(14) If $F''(X) = \sin X + e^{2X}$ and $F(0) = \frac{1}{4}$, $F'(0) = \frac{1}{2}$ find $F(X)$

$F'(X) = -\cos X + \frac{1}{2}e^{2X} + C_1 \quad \therefore \frac{1}{2} = -\cos 0 + \frac{1}{2} + C_1 \quad \therefore C_1 = 1$

$F(X) = -\sin X + \frac{1}{4}e^{2X} + X + C_2$

$\because F(0) = \frac{1}{4} \therefore \frac{1}{4} = -\sin 0 + \frac{1}{4}e^0 + 0 + c_2 \quad \therefore C_2 = 0$

$\therefore F(X) = -\sin X + \frac{1}{4}e^{2X} + X$

Q(15) Y = 2X - cot X at the point which lies on the curve and its x-coordinate Equals $\frac{\pi}{4}$

When $X = \frac{\pi}{4} \quad \therefore Y = 2 \times \frac{\pi}{4} - \cot 45^\circ = \frac{\pi}{2} - 1 \;\therefore$ point is $\left(\frac{\pi}{4}, \frac{\pi}{2} - 1\right)$

Slope of tangent $\frac{dY}{dX} = 2 + \csc^2 X$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{2} - 1\right)$

$\therefore \frac{dY}{dX} = 2 + (\csc 45^\circ)^2 = 4 \quad \therefore$ slope of normal is $-\frac{1}{4}$

Equation of tangent is $\frac{Y - \frac{\pi}{2} + 1}{X - \frac{\pi}{4}} = 4$ Equation of normal is $\frac{Y - \frac{\pi}{2} + 1}{X - \frac{\pi}{4}} = -\frac{1}{4}$

Q(16) Using on of the integration techniques to find $\int \frac{e^X}{\csc X} dX$

$\int \frac{e^X}{\csc X} dX = \int e^X \sin X dX = I$

$I = -e^X \cos X - \int e^X \cos X dX$

$I = -e^X \cos X + e^X \sin X - \int e^X \sin X dX$

$I = -e^X \cos X + e^X \sin X - I$

$2I = e^X(-\cos X + \sin X)$

$\therefore I = \frac{1}{2} e^X(-\cos X + \sin X)$

D	I
e^X	$\sin X$
e^X	$-\cos X$

Q(17) Using one of the integration techniques to find $\int \frac{X}{\sec^2 X} dX$

$\int X \cos^2 X dX = \int X\left(\frac{1}{2} + \frac{1}{2}\cos 2X\right) dX = \frac{1}{2}\int (X + X\cos 2X) dX$

$\frac{1}{2}\left(\frac{X^2}{2} + \frac{1}{2} X \sin 2X - \int \frac{1}{2} \sin 2X dX\right)$

$= \frac{1}{2}\left(\frac{X^2}{2} + \frac{1}{2} X \sin 2X + \frac{1}{4} \cos 2X\right) + C$

D	I
X	$\cos 2X$
1	$\frac{1}{2} \sin 2X$

Q(18) If $Y^2 = 4 + 2\sin X \cos X$ prove that :

$YY'' + Y'^2 + 2Y^2 = 8$

$Y^2 = 4 + \sin 2X \quad \therefore 2YY' = 2\cos 2X \quad \therefore YY' = \cos 2X$

$Y'^2 + YY'' = -2\sin 2X$

$L.H.S = -2\sin 2X + 2(4 + \sin 2X) = 8$

Q(19) If the slope of a tangent equal $\sqrt{\dfrac{e^{Y}}{1-\cos^2 X}}$ and the curve passing the point $\left(\dfrac{\pi}{4}, 0\right)$ find the equation of the curve

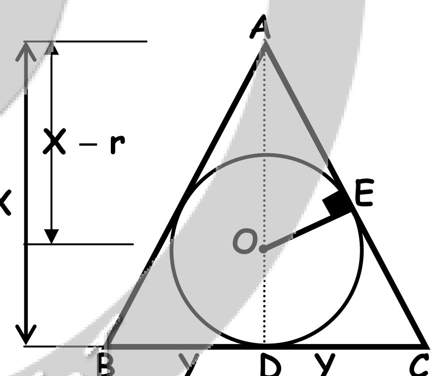
$$\frac{dY}{dX} = \frac{e^{\frac{1}{2}Y}}{\sqrt{1-\cos^2 X}} \quad \therefore\ e^{-\frac{1}{2}Y}dY = \frac{dX}{\sqrt{1-\cos^2 X}} \quad \therefore \int e^{-\frac{1}{2}Y}dY = \int \frac{dX}{\sqrt{1-\cos^2 X}}$$

$$\therefore \int e^{-\frac{1}{2}Y}dY = \int \frac{dX}{\sin X} \quad \therefore \int e^{-\frac{1}{2}Y}dY = \int \csc X dX \quad \therefore -2e^{-\frac{1}{2}} = -\ln\left|\csc X + \cot X\right|$$

$$2e^{-\frac{1}{2}Y} = \ln\left|\csc X + \cot X\right| + C$$

$$C = -\ln\left|\sqrt{2}+1\right|$$

Q(20) Find the smallest possible area of an isosceles triangle that is circumscribed about a circle of radius r



$$\Delta ADC \quad \Delta AEO \quad \therefore \frac{AE}{AD} = \frac{EO}{DC} = \frac{AO}{AC}$$

$$AE = \sqrt{(x-r)^2 - r^2} = \sqrt{X^2 - 2Xr}$$

$$\frac{AE}{AD} = \frac{EO}{DC} \therefore \frac{\sqrt{X^2-2Xr}}{X} = \frac{r}{Y} \quad \therefore Y = \frac{Xr}{\sqrt{X^2-2Xr}}$$

$$A = XY = \frac{X^2 r}{\sqrt{X^2-2Xr}} \quad \therefore A^2 = \frac{X^4 r^2}{X^2-2Xr}$$

$$A' = \frac{4X^3r^2\left(X^2-2Xr\right) - (2X-2r)\left(X^4r^2\right)}{\left(X^2-2Xr\right)^2} = 0$$

$$4X^3r^2\left(X^2-2Xr\right) - (2X-2r)\left(X^4r^2\right) = 0$$

$$4X^5r^2 - 8X^4r^3 - 2X^5r^2 + 2X^4r^3 = 0$$

$$2X^5r^2 - 6X^4r^3 = 0 \quad \therefore 2X^4r^2(X-3r) = 0 \quad \therefore X = 3r$$

$$\therefore Y == \frac{3r \times r}{\sqrt{9r^2 - 6r^2}} = \frac{3}{\sqrt{3}}r \quad \therefore A = 3r \times \frac{3}{\sqrt{3}}r = 3\sqrt{3}r^2$$

Answer the following questions 20 questions **Model Answer**

From 1to 12 choose the correct answer

Q(1) The slope of the tangent to the curve $Y = 2\cot X + \sqrt{2}\sec X$ At $X = \frac{\pi}{4}$ is ……….

① 2 ② -2

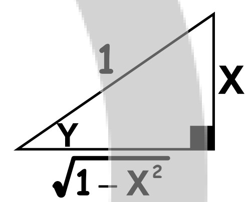
③ -1 ④ 1

$$\frac{dY}{dX} = -2\csc^2 X + \sqrt{2}\sec X \tan X \text{ at } X = \frac{\pi}{4} \quad \therefore \frac{dY}{dX} = -2\times 2 + \sqrt{2}\times\sqrt{2}\times 1 = -2$$

Q(2) If $X = \sin Y$ then $\frac{dY}{dX} = \ldots$

① $\frac{1}{1+X^2}$ ② $\frac{1}{\sqrt{X^2-1}}$

③ $\frac{1}{1-X^2}$ ④ $\frac{1}{\sqrt{1-X^2}}$



$$1 = \frac{dY}{dX}\cos Y \quad \therefore \frac{dY}{dX} = \frac{1}{\cos Y} = \sec Y = \frac{1}{\sqrt{1-X^2}}$$

Q(3) If F(X)=cotX then $F''\left(\frac{\pi}{4}\right)$ equals :

① $\frac{4}{9}$ ② -4

③ 4 ④ $\frac{9}{2}$

$$F'(X) = -\csc^2 X \quad \therefore F''(X) = -2\csc^2 X \cot X = -4$$

Q(4) A cube of ice melts preserving its shape at rate $1cm^3$/sec then the rate of change of the cube edge length when its volume is $8cm^3$

① $\frac{1}{12}$ ② $\frac{1}{6}$

③ $-\frac{1}{12}$ ④ $-\frac{1}{6}$

$$V = X^3 = 8 \quad \therefore X = 2 \quad \frac{dV}{dt} = 3X^2\frac{dX}{dt} \quad \therefore -1 = 12\frac{dX}{dt} \quad \therefore \frac{dX}{dt} = -\frac{1}{12}$$

Q(5) The region in the first quadrant bounded by the graph of $Y = \sec X$, $X = \frac{\pi}{4}$ and the axes is rotated about the X-axis then the volume generated =……..

① $\frac{\pi^2}{4}$ ② π

③ $\pi - 1$ ④ 2π

$$V = \pi\int_0^{\frac{\pi}{4}} (\sec X)^2 dX = \pi[\tan X]_0^{\frac{\pi}{4}} = \pi[\tan 45^\circ - \tan 0^\circ] = \pi$$

Q(6) If $\frac{d}{dX}\log_3(X^2+1)^{\frac{1}{3}} = \frac{2X}{a(X^2+1)}$ then a=……

① $3\ln 3$ ② 3

③ $\frac{1}{3\ln 3}$ ④ $\ln 3$

$$\log_3(X^2+1)^{\frac{1}{3}} = \frac{1}{3}\log_3(X^2+1) = \frac{1}{3\ln 3}\ln(X^2+1) \quad \therefore \frac{d}{dX}\frac{1}{3\ln 3}\ln(X^2+1) = \frac{1}{3\ln 3}\frac{2X}{(X^2+1)}$$

Q(7) If the curve $F(X) = X^3 + KX^2 + 4$, $K \in R$ has inflection point When X=2 then K=…..

① -6 ② 6

③ -3 ④ 9

$F'(X) = 3X^2 + 2XK \;\therefore F''(X) = 6X + 2K = 0$ when $X = 2 \;\therefore 2K = -12$

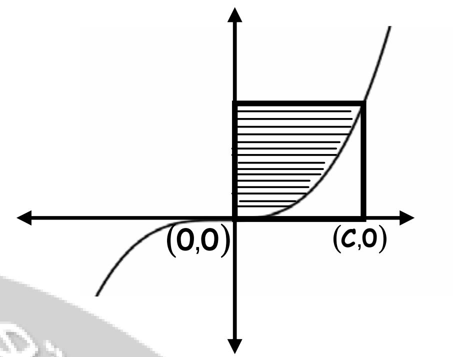
Q(8) If $F(X) = \cos^2 X$ then $F''(\pi) =$

① -2 ② 1

③ -2 ④ -1

$F'(X) = -2\cos X \sin X = -\sin 2X \therefore F'(X) = -2\cos 2X = -2\cos 360^\circ = -2$

Q(9)In the given figure: $F(X)=X^3$ the ratio between the area of the rectangle and the shaded part equals



① 3:4 ② 5:4

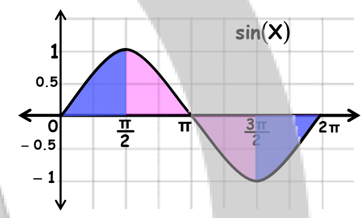
③ 4:3 ④ 4:5

$$A(\text{rectangle}) = C \times C^3 = c^4$$

$$\text{shaded part} = c^4 - \int_0^c X^3 dX$$

$$= c^4 - \left[\frac{X^4}{4}\right]_0^c = c^4 - \frac{c^4}{4} = \frac{3}{4}c^4 \therefore \text{ratio} = \frac{c^4}{\frac{3}{4}c^4} = \frac{4}{3}$$

Q(10) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin X| dX = ...$



①0 ②π

③2 ④$\frac{2}{\pi}$

$$2\int_0^{\frac{\pi}{2}} \sin X dX = 2[-\cos X]_0^{\frac{\pi}{2}} = 2(-\cos 90^\circ + \cos^\circ 0) = 2$$

Q(11) $\int_e^{2e} \frac{1}{X} dX = ...$

① 1 ② ln2

③ 0 ④ e

$$[\ln X]_e^{2e} = \ln 2e - \ln e = \ln 2$$

Q(12) $\int_0^1 |2X-1|\, dX = ...$

①$\frac{1}{2}$ ②$-\frac{1}{2}$

③$-\frac{1}{4}$ ④$\frac{1}{4}$

$$\int_0^{\frac{1}{2}}(-2X+1)dX + \int_{\frac{1}{2}}^1 (2X-1)dX = \left[-X^2+X\right]_0^{\frac{1}{2}} + \left[X^2 - X\right]_0^{\frac{1}{2}} = \frac{1}{2}$$

Q(13) Find the volume of the solid generated by revolving a region bounded by the curve $Y = 4 - X^2$ and the straight line $2X + Y = 4$ a complete revolution

$$4 - X^2 = 4 - 2X \quad \therefore X^2 - 2X = 0 \quad \therefore X(X-2) = 0 \quad \therefore X = 0 \text{ or } X = 2$$

$$\int_0^2 \left(4 - X^2 - 4 + 2X\right) dX = \left[-\frac{X^3}{3} + X^2\right]_0^2 = -\frac{8}{3} + 4 = \frac{4}{3}$$

Q(14) Find $\int \cot X \csc^3 X dX$

$$\int \cot X \csc^3 X dX = \int \frac{\cos X}{\sin X} \times \frac{1}{\sin^3 X} dX = \int \frac{\cos X}{\sin^3 X} dX$$

Let $Z = \sin X \quad \therefore \frac{dZ}{dX} = \cos X \quad \therefore dX = \frac{dZ}{\cos X}$

$$\int \frac{\cos X}{\sin^3 X} dX = \int \frac{\cos X}{Z^3} \times \frac{dZ}{\cos X} = \int Z^{-3} dZ = \frac{Z^{-2}}{-2} + C = -\frac{1}{2} \times \frac{1}{\sin^2 X} + C$$

Q(15) $\int_0^{\frac{\pi}{4}} X \sin 4X$

D	I
X	$\sin 4X$
1	$-\frac{1}{4}\cos 4X$

$$\int_0^{\frac{\pi}{4}} X \sin 4X = -\frac{1}{4} X \cos 4X + \frac{1}{4} \int \cos 4X dX$$

$$= \left[-\frac{1}{4} X \cos 4X + \frac{1}{16} \sin 4X\right]_0^{\frac{\pi}{4}}$$

$$= \left[-\frac{1}{4} \times \frac{\pi}{4} \cos \pi + \frac{1}{16} \times \sin \pi\right] - \left[-\frac{1}{4} \times 0 \cos 0 + \frac{1}{16} \times \sin 0\right] = \frac{\pi}{16}$$

Q(16) Find $\frac{dY}{dX}$ fore the curve $e^{Y}\ln(X+Y)=\cos(XY)$ at the point (1,0)

$$e^{Y}\frac{dY}{dX}\ln(X+Y)+\frac{1+\frac{dY}{dX}}{X+Y}e^{Y}=-\left(Y+X\frac{dY}{dX}\right)\sin XY \quad \text{at} \quad (1,0)$$

$$e^{0}\frac{dY}{dX}\ln(1)+\frac{1+\frac{dY}{dX}}{1+0}e^{1}=-\left(0+X\frac{dY}{dX}\right)\sin 0 \quad \therefore \frac{1+\frac{dY}{dX}}{1+0}e=0$$

$$1+\frac{dY}{dX}=0 \quad \therefore \frac{dY}{dX}=-1$$

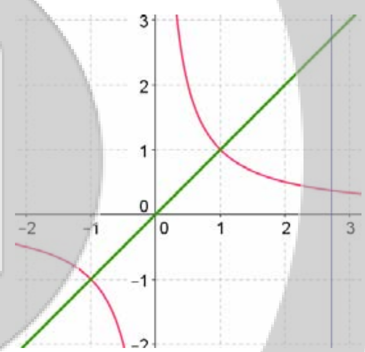
Q(17) Find the area bounded between the curve $Y=\frac{1}{X}$ and X=e

The line Y=X and the X axis

$$\frac{1}{X}=X \quad \therefore X^{2}=1 \quad \therefore X=\pm 1$$

$$\int_{0}^{1}X\,dX+\int_{1}^{e}\frac{1}{X}dX=\left[\frac{X^{2}}{2}\right]_{0}^{1}+\left[\ln X\right]_{1}^{e}$$

$$=\frac{1}{2}+[\ln e-\ln 1]=\frac{1}{2}+1=\frac{3}{2}$$



Q(18) If $\sin Y+\cos 2X=0$ prove that :

$$\frac{d^{2}Y}{dX^{2}}-\left(\frac{dY}{dX}\right)^{2}\tan Y=4\cos 2X\sec Y$$

$$\frac{dY}{dX}\cos Y-2\sin 2X=0 \quad \therefore \frac{dY}{dX}=\frac{2\sin 2X}{\cos Y}$$

$$\therefore \frac{d^{2}Y}{dX^{2}}\cos Y-\left(\frac{dY}{dX}\right)^{2}\sin Y-4\cos 2X=0$$

$$\therefore \frac{d^{2}Y}{dX^{2}}\cos Y-\left(\frac{dY}{dX}\right)^{2}\sin Y=4\cos 2X \qquad \div \cos Y$$

$$\therefore \frac{d^{2}Y}{dX^{2}}\cos Y-\left(\frac{dY}{dX}\right)^{2}\tan Y=4\cos 2X\sec Y$$

Q(19) If $Y=\sqrt{\dfrac{X^2+1}{X^2-1}}$, prove that : $(X^4-1)Y' + 2XY = 0$

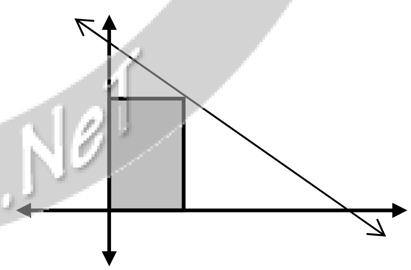
$Y=\left(\dfrac{X^2+1}{X^2-1}\right)^{\frac{1}{2}}$ $\quad \therefore \ln Y=\dfrac{1}{2}\ln\dfrac{X^2+1}{X^2-1}=\dfrac{1}{2}\left(\ln(X^2+1)-\ln(X^2-1)\right)$

$\therefore \dfrac{Y'}{Y}=\dfrac{1}{2}\left(\dfrac{2X}{X^2+1}-\dfrac{2X}{X^2-1}\right)=\dfrac{1}{2}\left(\dfrac{2X^3-2X-2X^3-2X}{X^4-1}\right)=\dfrac{1}{2}\left(\dfrac{-4X}{X^4-1}\right)$

$\therefore \dfrac{Y'}{Y}=\dfrac{1}{2}\left(\dfrac{-4X}{X^4-1}\right)$ $\quad \therefore \dfrac{Y'}{Y}=\dfrac{-2X}{X^4-1}$

$\therefore (X^4-1)Y' + 2XY = 0$

Q(20) A rectangle lies in the first quadrant with one vertex at the origin and two sides along the coordinates axes if the fourth vertex lies on the line X+2Y-10=0 find maximum area of this rectangle



$A=10-2Y$

$A=XY$

$A=10Y-2Y^2$

$A'=10-4Y=0 \quad \therefore Y=2.5 \quad \therefore X=5$

$A''=-4<0$

$A=5\times 2.5=12.5\ \text{units}^2$

Answer the following questions | 20 questions | **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $g(0) = 3$ and $g'(0) = 2$ $F(X) = g(X) + \frac{X}{g(X)}$ then $F'(0) = \ldots\ldots\ldots$

① $\frac{7}{3}$ ② $\frac{20}{9}$

③ 1 ④ $\frac{3}{2}$

$$F'(X) = g'(X) + \frac{1 \times g'(X) + Xg(X)}{[g(X)]^2} = 2 + \frac{2}{9} = \frac{20}{9}$$

Q(2) If $Y = 4n^3 + 4$, $Z = 3n^2 - 2$ then $\frac{dZ}{dY} = \ldots$

① $2n$ ② 2

③ $\frac{1}{2n}$ ④ 4

$$\frac{dY}{dn} = 12n^2 \quad \frac{dZ}{dN} = 6n \quad \therefore \frac{dZ}{dY} = \frac{dY}{dN} \times \frac{dN}{dZ} = 6n \times \frac{1}{12n^2} = \frac{1}{2n}$$

Q(3) $\frac{d}{dX} 5^{2X} = \ldots\ldots$

① 5^{2X} ② $(2X)5^{2X-1}$

③ 5 ④ $(\ln 25)(5^X)^2$

$$\frac{d}{dX} 5^{2X} = 5^{2X} \times 2\ln 5 = (\ln 25)(5^X)^2$$

Q(4) If $\ln(X + 4 + e^{-3X})$ then $F'(0) = \ldots\ldots.$

① $\frac{-2}{5}$ ② $\frac{1}{5}$

③ $\frac{1}{4}$ ④ $\frac{2}{5}$

$$F'(X) = \frac{1 - 3e^{-3X}}{X + 4 + e^{-3X}} \text{ at } X = 0 \quad F'(0) = \frac{1 - 3e^0}{0 + 4 + e^0} = -\frac{2}{5}$$

Q(5) If the function $F(X) = (X - a)^3 + 5$ has an inflection point at X=2 then a=

① 2 ② 6

③ 3 ④ 2

$F' = 3(X - a)^2 \quad \therefore F'' = 6(X - a) = 0 \quad \therefore X = 2$

Q(6) If $F(X) = \ln \sin X - \ln \cos X$ then $F'\left(\frac{\pi}{4}\right) = \ldots$

① -1 ② -2

③ 1 ④ 2

$F(X) = \ln \frac{\sin X}{\cos X} = \ln \tan X \quad \therefore F'(X) = \frac{\sec^2 X}{\tan X} \quad \therefore F'(45^\circ) = \frac{\sec^2 45^\circ}{\tan 45^\circ} = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

Q(7) A point is moving on the curve $Y = X^3$ such that $\frac{dX}{dt} = 2cm/sec$ then the rate of change of the slope of tangent at X=1

① 3 ② 6

③ 24 ④ 12

$\frac{dY}{dX} = 3X^2 \quad \therefore m = 3X^2 \quad \therefore \frac{dm}{dt} = 6X\frac{dX}{dt} = 6 \times 1 \times 2 = 12$

Q(8) If $Y^2 - 2\sqrt{X} = 0$ then $\frac{dY}{dX} =$

① $\frac{2Y}{\sqrt{X}}$ ② $\frac{X}{Y^2}$

③ $\sqrt{X}$ ④ $\frac{1}{Y^3}$

$Y^2 = 2\sqrt{X} \quad \therefore 2Y\frac{dY}{dX} = 2\frac{1}{2\sqrt{X}} = \frac{1}{\sqrt{X}} \quad \therefore \frac{dY}{dX} = \frac{1}{2Y\sqrt{X}} = \frac{1}{Y^3}$

Q(9) If $Y = 4\sec^2 X$ then $Y'\left(\frac{\pi}{4}\right)$ equals

① -8 ② 16

③ 0 ④ $4\sqrt{2}$

$Y' = 8\sec^2 X \tan X$ at $X=\frac{\pi}{4}$ $\therefore Y' = 8\sec^2 45° \tan 45° = 16$

Q(10) If $F(X) = 2\sin\frac{X}{2}\cos\frac{X}{2}$ then the 1000^{th} derivative equals

① sinX ② cosX

③ -sinX ④ -cosX

$Y^{2n} = (-1)^n \sin X$ fore even $Y^{2n+1} = (-1)^n \cos X$ fore odd

$\frac{d^{1000}y}{dX^{1000}} = (-1)^{500}\sin X = \sin X$

Q(11) If $\int_1^4 ax^2 dX = 42$ then a=….

① 2 ② 3

③ 4 ④ 5

$\left[\frac{aX^3}{3}\right]_1^4 = \frac{64a}{3} - \frac{a}{3} = 42 \quad \therefore \frac{63a}{3} = 42 \quad \therefore a = 2$

Q(12) If $\int_0^{\frac{\pi}{2}} 5^{\cos X}\sin X dX =$ ……

① 4 ② $\frac{-4}{\ln 5}$

③ $\frac{4}{\ln 5}$ ④ $\frac{1}{\ln 5}$

$\left[-\frac{1}{\ln 5}5^{\cos X}\right]_0^{\frac{\pi}{2}} = -\frac{1}{\ln 5}\times 5^{\cos 90°} + \frac{1}{\ln 5}\times 5^{\cos 0} = -\frac{1}{\ln 5} + \frac{5}{\ln 5} = \frac{4}{\ln 5}$

Q(13) Find $\int \frac{(\ln X)^2}{X} dX$

Let $\ln X = Z \quad \therefore \frac{dZ}{dX} = \frac{1}{X} \quad \therefore dX = XdZ$

$\int \frac{Z^2}{X} XdZ = \int Z^2 dZ = \frac{1}{3}Z^3 + C = \frac{1}{3}(\ln X)^3 + C$

Q(14) If $Y = e^{aX}$ find the value of a which satisfies $Y'' - 5Y' + 6Y = 0$

$Y' = ae^{aX} \quad , \quad Y'' = a^2e^{aX}$

$\therefore a^2e^{aX} - 5ae^{aX} + 6e^{aX} = 0 \qquad \div e^{aX}$

$\therefore a^2 - 5a + 6 = 0 \qquad \therefore a = 3 \text{ or } a = 2$

Q(15) If $Y = e^{3X} + X^2$ prove that : $\frac{d^2Y}{dX^2} - 2 = 9(Y - X^2)$

$\frac{dY}{dX} = 3e^{3X} + 2X \qquad \therefore \frac{d^2Y}{dX^2} = 9e^{3X} + 2$

$L.H.S = 9e^{3X} + 2 - 2 = 9e^{3X}$

$R.H.S = 9(e^{3X} + X^2 - X^2) = 9e^{3X}$

Q(16) Find the equation of the tangent to the curve
$e^{Y}\sin X + X - XY = \pi$ at the point $(\pi, 0)$

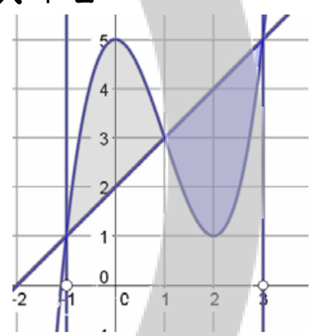
$e^{Y}Y'\sin X + e^{Y}\cos X + 1 - (XY' + Y) = 0$ when $X = \pi$ and $Y = 0$

$e^{0}Y'\sin\pi + e^{0}\cos\pi + 1 - (\pi Y' + 0) = 0$

$2 = \pi Y' \quad \therefore Y' = \frac{2}{\pi}$

$\frac{Y}{X - \pi} = \frac{2}{\pi} \quad \therefore \pi Y = 2X - 2\pi$

Q(17) Find the area of the region bounded by the curve of the two functions f, and g where: $f(x) = X^3 - 3X^2 + 5$, $g(x) = X + 2$



$X^3 - 3X^2 + 5 = X + 2 \quad \therefore X^3 - 3X^2 + 5 - X - 2 = 0$

$X^3 - 3X^2 + 3 - X = 0 \quad \therefore X^2(X - 3) - (X - 3) = 0$

$(X - 3)(X^2 - 1) = 0 \quad \therefore (X - 3)(X - 1)(X + 1)$

$$A = \int_{-1}^{1}\left[(X^3 - 3X^2 + 5) - (X + 2)\right]dX + \int_{1}^{3}\left[(X + 2) - (X^3 - 3X^2 + 5)\right]dX$$

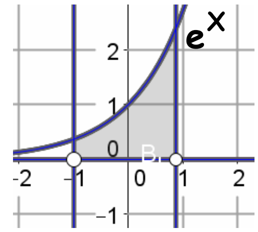
$$A = \int_{-1}^{1}(X^3 - 3X^2 - X + 3)\,dX + \int_{1}^{3}(-X^3 + 3X^2 + X - 3)\,dX$$

$$= \left[\frac{X^4}{4} - X^3 - \frac{X^2}{2} + 3X\right]_{-1}^{1} + \left[-\frac{X^4}{4} + X^3 + \frac{X^2}{2} - 3X\right]_{1}^{3} = 4 + 4 = 8$$

Q(18) Find the volume of the solid generated by revolving a region bounded by the two curves $Y = \frac{1}{X}$ and $X = 1$, $X = 4$ Y=0 a complete revolution about X-axis.

$$V = \pi\int_{1}^{4}\left[\frac{1}{X}\right]^{2}dX = \frac{3}{4}\pi$$

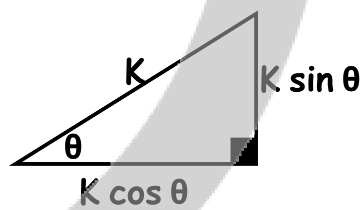
Q(19)) If the volume generated by revolution of the shaded par around X-axis from X=-1 and X=K equals $\frac{\pi}{2}\left(e^{10}-e^{-2}\right)$ find K



$$\pi\int_{-1}^{K}\left(e^{X}\right)^{2}dX=\pi\int_{-1}^{K}e^{2X}dX=\pi\left[\frac{1}{2}e^{2X}\right]_{-1}^{K}$$

$$=\frac{\pi}{2}\left(e^{2K}-e^{-2}\right)=\frac{\pi}{2}\left(e^{10}-e^{-2}\right)\quad\therefore e^{2K}=e^{10}\quad\therefore 2K=10\quad\therefore K=5$$

Q(20) Find the greatest perimeter of a right triangle with hypotenuse of K cm



$$P=K+K\cos\theta+K\sin\theta$$

$$\frac{dP}{d\theta}=-K\sin\theta+K\cos\theta$$

$$-K\sin\theta+K\cos\theta=0\quad\therefore\cos\theta=\sin\theta$$

$$\frac{\sin\theta}{\cos\theta}=\tan\theta=1\quad\therefore\ \theta=45^{\circ}$$

$$P=K+\sqrt{2}K$$

Calculus Exam 2017 model **(10)** page(1)

Answer the following questions | 20 questions | **Model Answer**

From 1to 12 choose the correct answer

Q(1) If $\cos 2X = \sin Y$ then $\frac{dY}{dX} = ...$ at $X = \frac{\pi}{6}$

① 2 ① -1

③ -2 ③ 1

$-2\sin 2X = \frac{dY}{dX}\cos Y \quad \because \sin Y = \cos 60^o \quad \therefore Y = \frac{\pi}{6}$

$\therefore -2\sin 60^o = \frac{dY}{dX}\cos 30^o \quad \therefore \frac{dY}{dX} = -2$

Q(2) The equation of the normal to the curve $Y = X + \sin X \cos X$

At $X = \frac{\pi}{2}$

① $X = \pi$ ② $X = 2$

③ $X + \pi = 0$ ④ $X = \frac{\pi}{2}$

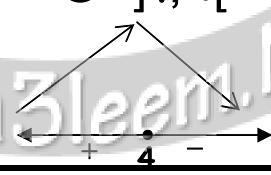
$Y = x + \frac{1}{2}\sin 2X \quad \therefore \frac{dY}{dX} = 1 + \cos 2X \quad$ when $X = \frac{\pi}{2} \quad \therefore \frac{dY}{dX} = 0 \quad \therefore \quad X = \frac{\pi}{2}$

Q(3) The function $F(X) = e^{8X - x^2}$ increasing on

① $]-\infty, 0[$ ② $]-\infty, 4[$

③ $]0, \infty[$ ④ $]4, \infty[$

$F^/(X) = (8 - 2X)e^{8X - x^2} \quad \therefore 8 - 2X = 0 \quad \therefore X = 4$



increase $]-\infty, 4[$

Q(4) If $y = \ln(X-2)^2$ then $Y^/(3) = ...$

① $\frac{1}{3}$ ② $\frac{4}{3}$

③ 2 ④ 1

$Y^/ = \frac{2}{X-2}$ at X=3 $Y^/ = \frac{2}{3-2} = 2$

Q(5) $\int_0^{\frac{\pi}{2}} \frac{(\sin X + \cos X)^2}{\sqrt{1+\sin 2X}} dX =$

① 0 ② 1

③ 2 ④ 3

$$\int_0^{\frac{\pi}{2}} \frac{(\sin X+\cos X)^2}{\sqrt{\sin^2 X+\cos^2 X+2\sin X\cos X}} dX = \int_0^{\frac{\pi}{2}} \frac{(\sin X+\cos X)^2}{\sqrt{(\sin X+\cos X)^2}} dX = \int_0^{\frac{\pi}{2}} (\sin X+\cos X) dX = 2$$

Q(6) If $F(X) = \begin{vmatrix} X & X^2 & X^3 \\ 1 & 2X & 3X^2 \\ 0 & 2 & 6X \end{vmatrix}$ then $F'(X) =$

① 12 ② 6X

③ $2X^3$ ④ $6X^3$

$$F(X) = \begin{vmatrix} X & X^2 & X^3 \\ 1 & 2X & 3X^2 \\ 0 & 2 & 6X \end{vmatrix} = X\begin{vmatrix} 1 & X & X^2 \\ 1 & 2X & 3X^2 \\ 0 & 2 & 6X \end{vmatrix} = X\begin{vmatrix} 1 & X & X^2 \\ 0 & X & 2X^2 \\ 0 & 2 & 6X \end{vmatrix} = X(6X^2 - 4X^2) = 2X^3$$

Q(7) If the graph of $Y = F(X)$ contains the point (0,2) , $\frac{dY}{dX} = \frac{-x}{Ye^{X^2}}$

And $F(x) > 0$ For all X then $F(X) = \ldots$

① $\sqrt{3+e^{-X^2}}$ ② $\sqrt{3+e^{X^2}}$

③ $\sqrt{3}+e^{-X}$ ④ $3+e^{-X^2}$

$\frac{dY}{dX} = \frac{-x}{Ye^{X^2}}$ $\therefore YdY = -Xe^{-X^2}dX$ $\therefore \frac{Y^2}{2} = \frac{1}{2}e^{-X^2} + C$ $\therefore C = \frac{3}{2}$

$\therefore \frac{Y^2}{2} = \frac{1}{2}e^{-X^2} + \frac{3}{2}$ $\therefore Y^2 = e^{-X^2} + 3$

Q(8) The derivative of X^6 with respect to X^3 is …….

① $6X^6$ ② $2X^3$

③ $3X^2$ ④ X^2

Let $U = X^6$ $\therefore \frac{dU}{dX} = 6X^5$ $V = X^3$ $\therefore \frac{dV}{dX} = 3X^2$

$\frac{dU}{dV} = \frac{dU}{dX} \times \frac{dX}{dV} = 6X^5 \times \frac{1}{3X^2} = 2X^3$

Q(9) If $Y = \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$ then $\frac{dY}{dX} = ...$ when $\theta = \frac{3\pi}{4}$

① -2 ② 3

③ 2 ④ 4

$Y = \sqrt{\frac{1+2\cos^2\theta - 1}{1-1+2\sin^2\theta}} = \cot\theta \quad \therefore \frac{dY}{dX} = -\csc^2\theta = -\csc^2\frac{3\pi}{4} = -2$

Q(10) $\frac{d}{dX}X^X =$

① XX^{X-1} ② $X^X(\ln X + 1)$

③ $X^X \ln X$ ④

$Y = X^X \quad \therefore \ln Y = X\ln X \quad \therefore \frac{Y'}{Y} = \ln X + 1 \quad \therefore Y' = X^X(\ln X + 1)$

Q(11) If $\int \frac{5^{\frac{1}{X}}}{X^2} dX = K5^{\frac{1}{X}}$ then K=....

① $\ln 5$ ② $\frac{1}{\ln 5}$

③ $-\ln 5$ ④ $-\frac{1}{\ln 5}$

$-\ln 5$

Q(12) If $F(X) = \log_{X^2}(\ln X)$ then $F'(e) =$

① 0 ② $\frac{1}{e}$

③ 1 ④ $\frac{1}{2e}$

$\log_{X^2}(\ln X) = \frac{\ln\ln X}{\ln X^2} = \frac{\ln\ln X}{2\ln X} \quad \therefore F'(X) = \frac{1}{2}\left(\frac{\frac{1}{X\ln X}\times\ln X - \frac{1}{X}\ln\ln X}{(\ln X)^2}\right) = \frac{1}{e}$

Q(13) Find the volume of the solid generated by revolving the plane region bounded by the curves : $y = X^3 + 1$, y = 0 and X = 0, X = 1 a complete revolution about X-axis.

$$V = \pi\int_0^1 (X^3+1)^2 dX = \pi\int_0^1 (X^6+2X^3+1)dX = \pi\left[\frac{X^7}{7}+\frac{2X^4}{4}+X\right]_0^1$$

$$= \pi\left[\frac{1}{7}+\frac{1}{2}+1\right] = \frac{32}{14}\pi$$

Q(14) A cuboid of metal whose base is square .if the side length of the base increase at rate of 0.4cm/sec and the height decrease at a rate of 0.5m/sec find the rate of change of the volume when the side length of the base is 6cm and the height is 5cm

let the dimensions of the cuboid X , X and Y

$$\frac{dX}{dt} = 0.4 \quad , \frac{dY}{dt} = -0.5$$

$$\because V = X^2Y$$

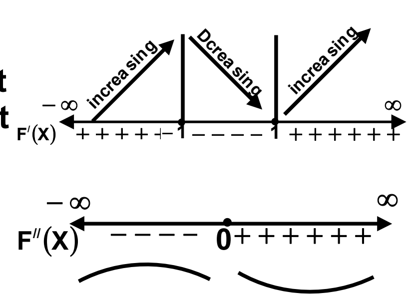
$$\therefore dVdt = X^2\frac{dX}{dt} + Y \times 2X\frac{dX}{dt}$$

$$= 6^2 \times -0.5 + 5 \times 2 \times 6 \times 0.4 = 6cm^3 / sec$$

Q(15) Determine The intervals of increasing and decreasing of the function : $Y = X^3 - 3X + 4$, then sketch its curve indicating on it its points of local maximum , local minimum and the point of inflection if exist

$F'(X) = 3X^2 - 3 = 0 \quad \therefore X = \pm 1 \quad \therefore$ critical points $(1,2), (-1,6)$

$F''(X) = 6X \quad \because \begin{cases} F''(1) = 6(+ve) & \therefore (1,2) \text{ Local maximum point} \\ F''(-1) = -6(+ve) & \therefore (-1,6) \text{ Local minimum point} \end{cases}$



Interval of increase $\rightarrow]1,\infty[,]-\infty,-1[$

Interval of decreasing $\rightarrow]-1,1[$

$F''(X) = 6X = 0 \quad \therefore X = 0 \quad , Y = 4$

The point (0,4) is an inflection point

Convex up interval $]-\infty,0[$ **Convex down interval** $]0,\infty[$

Q(16) Find the equation of the tangent to the curve

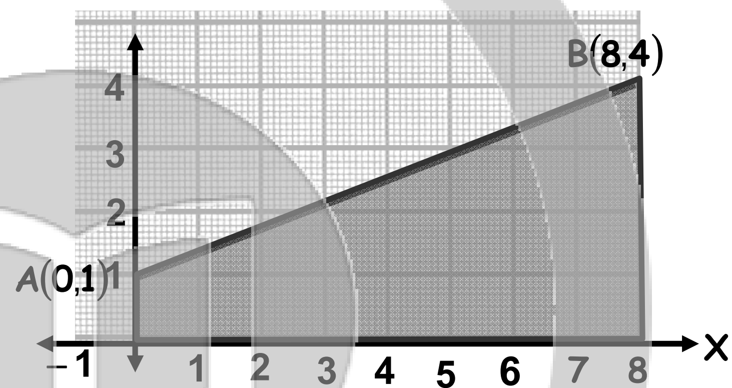
$Y = \log_e\left(2 - \sqrt{2}\cos X\right)$ at the point on it and its X-coordinate equals $\frac{\pi}{4}$

$\frac{dY}{dX} = \frac{\sqrt{2}\sin X}{2-\sqrt{2}\cos X}$ when $X = \frac{\pi}{4}$ $\therefore \frac{dY}{dX} = \frac{\sqrt{2}\sin 45^\circ}{2-\sqrt{2}\cos 45^\circ} = 1$

When $X = \frac{\pi}{4}$ $\therefore Y = \ln\left(2 - \sqrt{2}\cos 45^\circ\right) = \ln 2 - 1 = \ln 1 = 0$

$\therefore \frac{Y-0}{X-\frac{\pi}{4}} = 1$ $\therefore Y = X - \frac{\pi}{4}$

Q(17) In the given figure :



Find the volume generated by rotation AB about X-axis

slope of $\overleftrightarrow{AB} = \frac{4-1}{8} = \frac{3}{8}$

equation of $\overleftrightarrow{AB}$

$\frac{Y-1}{X-0} = \frac{3}{8}$ $\therefore 8Y - 8 = 3X$

$\therefore 8Y = 3X + 8$ $\therefore Y = \frac{1}{8}(3X+8)$

$V = \pi\int_0^8\left(\frac{1}{8}(3X+8)\right)^2 dX = \pi\frac{1}{64}\int_0^8\left(9X^2+64+48X\right)dX =$

$= \frac{\pi}{64}\left[3X^3 + 64X + 24X^2\right]_0^8 = 56\pi$

Q(18) Using one of the integration techniques to find

$\int(\cos X + \sec X)^2 dX$

$\int\left(\cos^2 X + \sec^2 X + 2\right)dX =$

$\int\left(\frac{1}{2}(1+\cos 2X) + \sec^2 X + 2\right)dX$

$= \frac{1}{2}\left(X + \frac{\sin 2X}{2}\right) + \tan X + 2x + C$

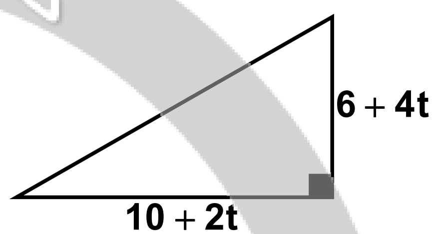
Q(19) The base of a right triangle increase with rate 2cm/sec and the height increase by rate 4cm/sec if the base and the height are originally 10cm , 6cm Find the time rate of change of the base angle when the base angle is 45°

$\tan\theta = \frac{6+4t}{10+2t} = \frac{3+2t}{5+t}$ $\therefore \sec^2\theta \times \frac{d\theta}{dt} = \frac{2(5+t)-(3+2t)}{(5+t)^2} = \frac{7}{(5+t)^2}$

when $\theta = 45^\circ$ $\therefore \tan 45^\circ = \frac{3+2t}{5+t} = 1$

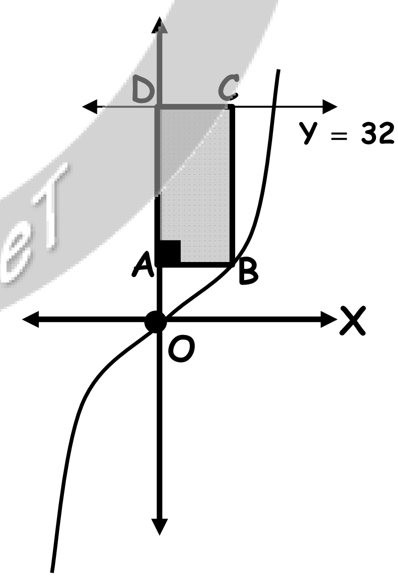
$3+2t = 5+t$ $\therefore$ $t = 2$ $\therefore \sec^2 45^\circ = \frac{1}{\cos^2 45^\circ} = 2$

$\therefore 2\times\frac{d\theta}{dt} = \frac{7}{7^2}$ $\therefore \frac{d\theta}{dt} = \frac{1}{14}$



Q(20) In the given figure :

$F(X) = X^3$ find the greatest area of rectangle ABCD



$A = X(32-Y) = X(32-X^3)$

$A = 32X - X^4$ $\therefore \frac{dA}{dX} = 32 - 4X^3$

$32 - 4X^3 = 0$ $\therefore 4X^3 = 32$ $\therefore X^3 = 8$ $\therefore X = 2$

$\frac{d^2A}{dX^2} = -12X^2 < 0$

$A = 32(2) - 2^4 = 48$