

Remember that

Probability:

- 1) Non accruing of A : $P(A') = 1 - P(A)$
- 2) accruing of A or B (**at least one** of them)
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- 3) accruing (**at most one** of them) $P(A \cap B)'$
- 4) **Only one** of them
 $P(A \cup B) - P(A \cap B) = P(A) + P(B) - 2P(A \cap B)$
- 5) **Only A** (accruing of A and non accruing of B)
 $P(A - B)$ Or $P(A \cap B')$
 $= P(A) - P(A \cap B)$
- 6) **Non accruing of both**
 $P(A' \cap B') = P(A \cup B)'$
- 7) $P(A \cup B)' = P(B - A)'$
- 8) A, B are **mutually exclusive**: $A \cap B = \emptyset$
 $\therefore P(A \cap B) = 0$
- 9) $A \subset B$ $\therefore P(A \cap B) = P(A)$
 $P(A \cup B) = P(B)$
- 10) A, B, C are is race
 $\therefore P(A) + P(B) + P(C) = 1$
 $A \cap B = A \cap C = B \cap C = \emptyset$
- 11) If $P(A) = P(A')$ then $P(A) = P(A') = \frac{1}{2}$
- 12) $P(A - B') = P(A \cap B)$
- 13) $P(A) \in [0, 1]$
- 14) $P(A' - B') = P(A' \cap B) = P(B) - P(A \cap B)$
- 15) Even: divisible by 2
 Odd: indivisible by 2
 Prime : 2, 3, 5, 7, 11, 13, 17, 19, 23,

* Random Variables

$$1) P(x_1) + P(x_2) + \dots = 1$$

$$\sum F(x) = 1$$

$$\sum P(x = r) = 1$$

$$2) \mu: \text{mean} = \sum x_r \cdot F(x_r)$$

$$\sigma^2: \text{Variance} = \sum x_r \cdot F(x_r) - \mu^2$$

$$\sigma: \text{S.D} = \sqrt{\sigma^2}$$

$$\text{Variation Coefficient} = \frac{\sigma}{\mu} \%$$

3) If μ is given and we want to find a constant

$$\therefore \text{Apply } 1) \sum f(x) = 1$$

$$2) x_i \cdot F(x) = \mu \leftarrow \text{given}$$

4) Density function

1st Draw the graph.

$$p(a < x < b) = \frac{1}{2} [p(b) + p(a)](b-a)$$

5) If the density function F is defined on the Interval $[a, b]$ then $p(a < x < d) = 1$

$$6) P(c < x < d) = \frac{1}{2} (f(d) + f(c)) (d - c) \text{ such that:}$$

$[c, d] \subset$ the interval of the definition.

$[c, d] \subset$ one rule of the two rules.

7) If the function defined by multi-rules as:

$$F(x) = \begin{cases} \text{rule(1)} & \text{when } a < x < c \\ \text{rule(2)} & \text{when } c < x < d \end{cases} \quad \text{then}$$

$$P(a < x < d) = p(a < x < c) + p(c < x < d)$$

• The Normal distribution

Standard (z)

- 1) $p(z > 0) = p(z < 0) = 0.5$
- 2) from the table $p(0 < z < k)$
- 3) Use the graph

Normal variable (x)

$$z = \frac{x - \mu}{\sigma}$$

percent = prob. \times 100%

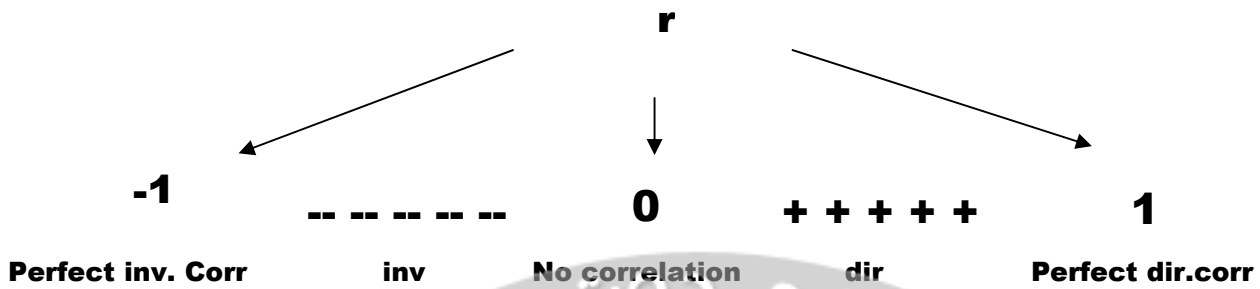
Number = prob. \times given no.

Use the **calculator** to
Find any operation

To evaluate any probability
The form must be in
 $p(0 < z < a)$
Where a is (+ve)

Do not forget the similarity about the line $x = \mu$

Correlation



Person Linear Correlation

$$1) r = \frac{n \sum xy - \sum x \cdot \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$$

2) Correlation Coefficient Between $X = x - ..$ & $Y = y - ..$ Does not change.

3) If $\sum (x-5) = a$, $\sum (y-3) = b$ then
 $\sum x = a + 5n$ & $\sum y = b + 3n$
 n : number of data

Spearman rank correlation Coefficient

$$1) r = 1 - \frac{6 \sum D^2}{n(n^2 - 1)}$$

2) The rank of frequency value = $\frac{\text{sum of ranks}}{\text{frequency}}$

3) $\sum D = \text{zero}$

4) The order of estimations

Weak, pass, good, very good, excellent

Regression

1) equation of regression of Y on X :

$Y = A X + B$ where:

A is the coefficient of regression of Y on X.

And given by:

$$A = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$B = \frac{\sum y - A \sum x}{n}$$

2) Equation of regression of X on Y :

$X = A Y + B$ where:

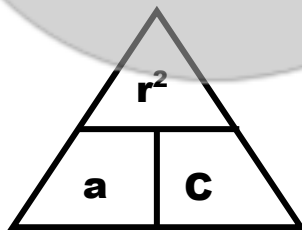
C is the coefficient of regression of X on Y

And given by:

$$C = \frac{n \sum xy - \sum x \sum y}{n \sum y^2 - (\sum y)^2}$$

$$D = \frac{\sum x - C \sum y}{n}$$

3)



Where, r : person's corr. Coeff.

a : coeff. Of reg. of y on x

c : coeff. Of reg. of x on y .

And each of a , c , r have the same sign.

4) $n = 7$, $\sum(x-3) = 0 \Rightarrow \sum x = 21$

5) If $x - \dots = X$, $y - \dots = Y$

The eq. of regression between x & y different of

The eq. of regression between Y & X .

Exercises

1) 3-persons A, B, C in a race if probability of A is twice that of B and probability of B is twice that of C

Find: $p(B)$, $p(A)$, $p(A \text{ or } B)$, $p(B \& C)$

2) A sample space S containing A, B, C Where

$$\frac{P(A')}{P(A)} = \frac{8}{3}, \frac{P(B')}{P(B)} = \frac{5}{2} \quad \text{Find } \frac{P(C')}{P(C)}$$

3) If the prob. that Aly solve a problem is $\frac{3}{4}$, the prob. that Rahmy

Solve that a problem is $\frac{2}{3}$, and that both of them solve the

problem is $\frac{1}{2}$ find

a) No solving of the problem

b) Rahmy only

c) Only one of them

d) At most one of them

4) Two sliders hit at a target the probability that the 1st hit the target is 0.5, second 0.7, both 0.35 find

a) Hitting the target

b) Only one of them

$$5) P(A) = \frac{5}{8}, \quad p(B) = \frac{1}{2}, \quad p(A \cap B) = \frac{1}{4}$$

Find

$$P(A \cup B)$$

$$P(A' \cap B')$$

$$P(A' \cup B')$$

$$P(A' - B)$$

$$P(A' - B')$$

$$P(A' \cup B)$$

$$P(A \cap B')$$

6) A Family has three children, write down the sample space with respect to the sex (boy or girl) and their order according to the age, given that there are no twins, Determine the Set representation of the following events

A: the family has two girls

B: the eldest child is a boy

7) The set $\{1, 2, 3, 4\}$ is used in writing a number Which consists of two different digits , find the probability of the following events :

A: the tens digit is even

B: the units digit is even

C: both digits are even

D: the units digit or the tens digit is even

8) $A, B \subset S$, $p(A) = 0.5$, $P(A \cup B) = 0.6$, find $P(B)$ in each case of the following:

a) A, B are two Mutually exclusive

b) $A \subset B$

c) $P(A \cap B') = 0.3$

9) Three persons A, B, C are in a swimming race A, B have the same probability of winning and each is twice as likely to win as C , find the probability that A, B win given that only one person will win.

10) A, B are two events of a sample space of A random experiment , $P(A) = 0.9$, $P(B) = 0.5$, and $P(A' \cup B') = 0.7$, find the probability of each of the following:

a) The occurrence of both A, B together.

b) The occurrence of only A .

c) The occurrence of at least one of the two events.

d) The occurrence of only one of the two events.

11) If x is random variable its probability distribution is

x	0	1	2	A	6
$F(x)$	0.1	0.1	0.3	B	.03

Find a, b where $\mu = 3.5$, then calculate the standard deviation of x .

12) Let x be a discrete random variable, its range is

$\{-1, 0, 1, 3\}$, and $p(x=-1) = p(x=2) = 0.3$, $p(x=0) = 0.2$, find

$p(x=1)$, and calculate the standard deviation of x .

13) If x is random variable its mean $\mu = 3$, and probability distribution is

x	0	2	k	4
$F(x)$	m	$2m$	$\frac{1}{3}$	$5m$

- a) find the value of m, k
 b) Calculate the coefficient of variation of x .

14) If x random variable its probability distribution is given by the function $f(x) = \frac{x^2 + 1}{a}$, $x = 0, 1, 2, 3$. Find:

- a) The value of a
 b) Calculate the coefficient of variation of x .

15) If x is a random variable its range $\{-1, 0, 1, 2\}$ and $P(x=r) = \frac{x+r}{18}$ for all $r \in$ its range, then find x and the mean and the standard deviation of x .

16) If x is a random variable its density function is

$$F(x) = \begin{cases} ax & 0 \leq x \leq s \\ 0 & \text{other..wise...find} \end{cases}$$

- 1) $p(1 < x < 3)$
 2) $p(x < 3)$
 3) $p(2 < x < 4)$

17) Let x be a continuous random variable with Density function

$$f(x) = \begin{cases} \frac{1}{6} \left(x + \frac{1}{2}\right) & 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Then find a) $p(1 \leq x \leq 2)$ b) $p(x \geq 2)$

18) Let x be a continuous random variable with density function

$$F(x) = \begin{cases} \frac{x-1}{k} & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Then find a) the value of k
b) $P(2 < x < 4)$

19) Let x be a continuous random variable with density function

$$i) F(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Then find the value of

a) $P(x < \frac{1}{2})$

b) $p(\frac{1}{2} < x < 1\frac{1}{2})$

$$ii) F(x) = \begin{cases} \frac{1}{4}(2+x) & -2 < x < 0 \\ \frac{1}{4}(2-x) & 0 < x < 2 \end{cases} \text{ then compute:}$$

$P(-1.5 < x < 1.5)$

$$iii) F(x) = \begin{cases} \frac{2}{9}(x+5) & k < x < k+3 \\ 0 & \text{otherwise} \end{cases} \text{ then find the value of } k$$

hence find $P(x < 7)$.

20) If z is a standard random variable, find

1) $p(0 \leq z \leq 1.54)$

2) $p(z > 1.25)$

3) $p(z < -1.4)$

4) $p(-1.5 < z < 0)$

5) $p(-2 < z < -1)$

6) $p(1.2 < z < 2)$

21) If z is a standard random variable, find the positive real number k which satisfies:

1) $p(-k < z < k) = 0.9030$	2) $p(-2 < z < k) = 0.9692$
3) $p(z > k) = 0.1170$	4) $p(z < k) = .9997$

22) Let x be a normal random variable with $\mu = 153$, $\sigma = 25$, find k where $p(x > k) = .1$, $p(x > k) = 0.06$

23) If the height of student are normally distributed , with mean 165 cm and variance 25 , a student is chosen at random from them . Find the probability that his height will be

- 1) Greater than 180
- 2) less than 160 cm
- 3) Between 160 cm and 170 cm

24) The monthly income of 200 families in a city is normally distribution with mean $\mu = 480$ L.E , and standard deviation $\sigma = 60$ L.E , a family is chosen at random from these families. find.

- 1) the probability that its income is more than 600 L.E
- 2) the percentage that its income is more than 660 L.E
- 3) the no. of families whose income is more than 600 L.E

25) If x is a random variable with mean μ , standard deviation σ , find :

- 1) $p(\mu - \sigma < x < \mu + \sigma)$
- 2) $p(\mu - 2\sigma < x < \mu + 2\sigma)$
- 3) $p(\mu - 3\sigma < x < \mu + 3\sigma)$

26) the rate of boys intelligence in a school are normally distributed with mean 104 , and standard deviation 3 , while the rates for girls are normally distributed with mean 102 and standard deviation 2 . , find :

- 1) the percentage o boys whose rates of intelligence are greater than the mean of girls rate of intelligence
- 2) the percentage o girls whose rates of intelligence are less than the mean of boys rate of intelligence

27) Suppose the marks for an examination are normally distributed with mean 75 and standard deviation 15, the top 15% of the students receive an excellent estimate. Find the least mark in order to the student get an excellent estimate.

28) If x is a normal random variable of mean μ , standard deviation $\sigma = 8$ and $p(x < 40) = 0.158$ find :

- the value of μ
- $p(x > 50)$

29) Compute Pearson's correlation coefficient for the following data on heights and weight of a group of 8-students:

Height	164	152	184	164	156	168	164	176
weight	52	40	60	52	42	50	52	60

Calculate spearman's rank correlation coefficient between the height and the weight for the same data.

30) The following table gives the scores of 8 students in history and geography, calculate the correlation coefficient to measure the degree of relationship between the marks in the two subjects.

History	weak	pass	pass	weak	v.good	exc	good	pass
geography	pass	Good	good	pass	good	v.good	exc	weak

31) A sample consists of 12 workers is selected, to study the relation between the income (in tens of pounds) y and the age (in years) x , the following data are obtained: $\sum x = 300$, $\sum y = 200$, $\sum x^2 = 8000$, $\sum y^2 = 3500$, $\sum xy = 5120$

Find the corr. Coef. Between the income and the age.

32) The following table shows the income and the consumption (in tens L.E) of 7 families in a certain city.

Income	38	32	42	48	40	44	50
consumption	24	21	27	30	27	33	36

Find: 1) the regression line of consumption on the income.

2) an estimate for the consumption when the income = 450 L.E.

33) A and B are two events in a sample space of a random experiment P if $P(A) = 0.45$ $P(B) = 4x$, and $P(A \cup B) = 10x - 0.1$, find the value of x in each of the following cases:

a) A, B are two mutually exclusive

b) $A \subset B$

c) $B \subset A$

d) $P(A \cap B) = \frac{11}{140}$



With my best wishes for you

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