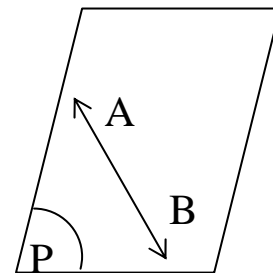


Lines and Planes

The Plane : is the surface where the line joining any two points
(The wall surface , the faces of a cube ,)



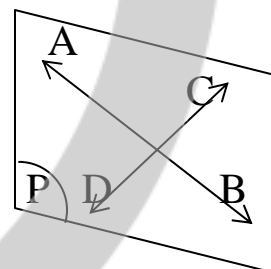
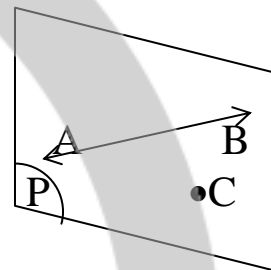
Notice:

- 1) The plane has no limit and it has no thickness
- 2) It divides the space in two parts

Determining a plane in the space :

The plane is determining by one of the following

- 1) Three distinct non – collinear points
- 2) line and a point does not belong to it
- 3) Two intersecting Lines
- 4) Any two parallel lines



Relative positions of a line w.r.t a plane

Let L be a line and P be a plane

- 1) $L \parallel \text{plane } P \quad L \cap P = \phi$

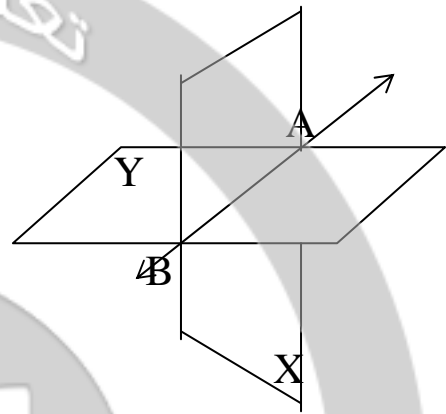
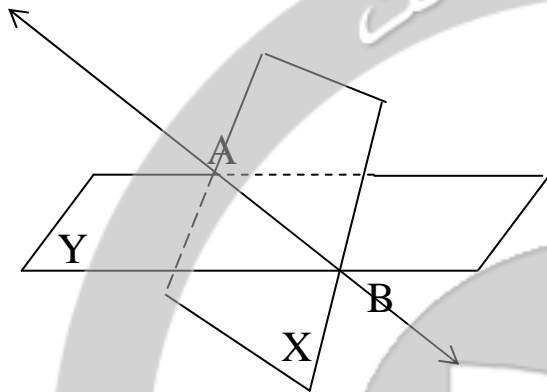
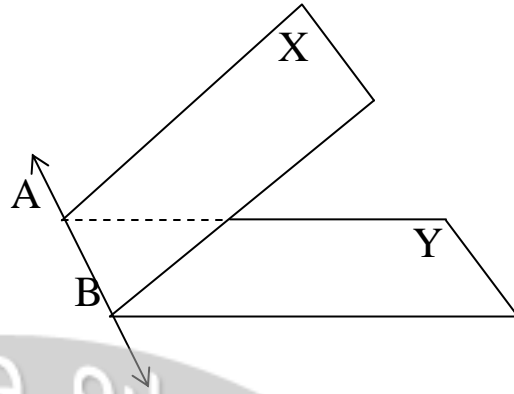
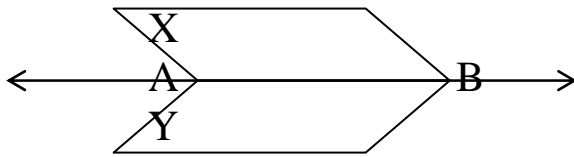
A diagram showing a horizontal line L above a horizontal plane P. A double-headed arrow above the line indicates its length. A vertical double-headed arrow between the line and the plane indicates the distance between them.
- 2) L intersects the plane in one point

A diagram showing a vertical line L intersecting a horizontal plane P at a single point.
- 3) L lies completely in the plane $L \subset P$

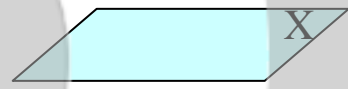
A diagram showing a horizontal line L lying on top of a horizontal plane P. A double-headed arrow above the line indicates its length.

Relative position of two planes

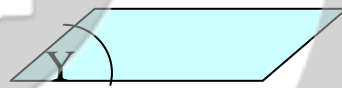
1) the two planes intersect



2) The two plane coincide :



3) The two plane are parallel
 $X \cap Y = \phi$

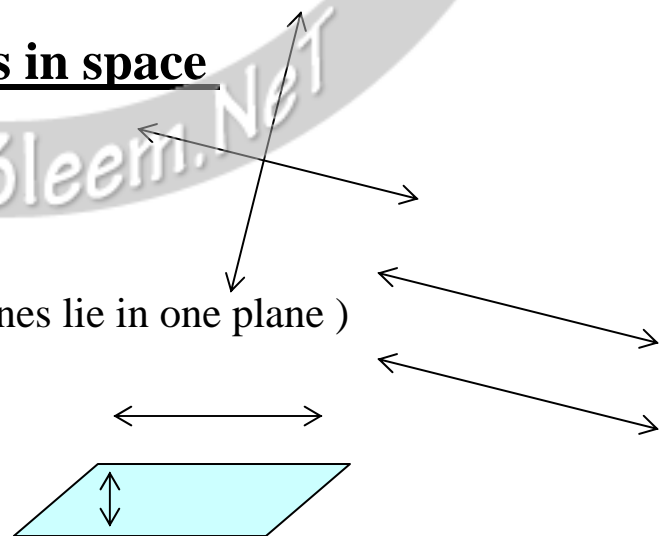


The relation between two lines in space

1) The two lines intersect

2) The two lines are parallel (the two lines lie in one plane)

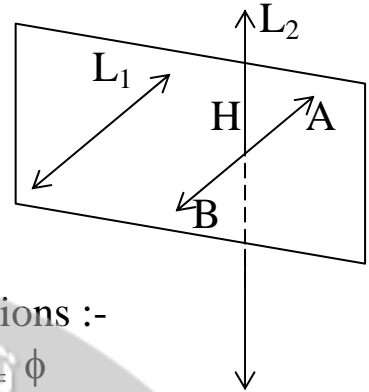
3) The two are **SKEW** (non-coplanar)



The angle between two Skew Lines

$L_1 \subset X, L_2 \subset X, L_2 \cap X = \{H\}$. to determine the angle between them
 Draw \overleftrightarrow{AB} passing through H such that $\overleftrightarrow{AB} \parallel L_2$

The angle included between L_2 and \overleftrightarrow{AB} is that included between L_1 and L_2

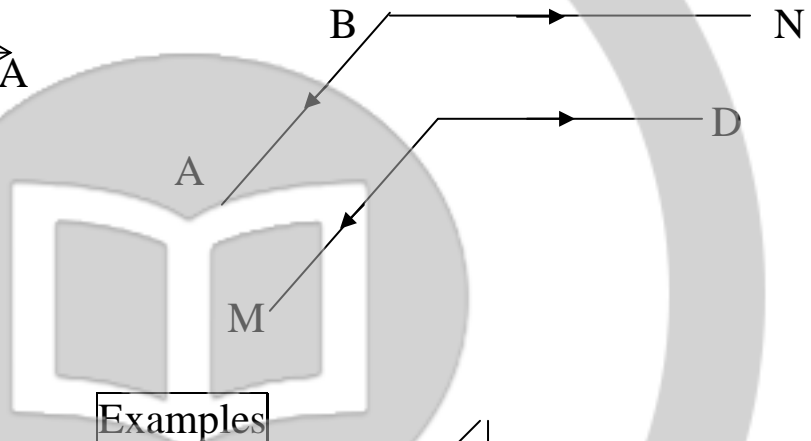


N.B : to prove that $L_1 \parallel L_2$, prove each of the conditions :-

- i) L_1 and L_2 are lying in one plane
- ii) $L_1 \cap L_2 = \phi$

Through two given Skew lines, can pass two parallel planes

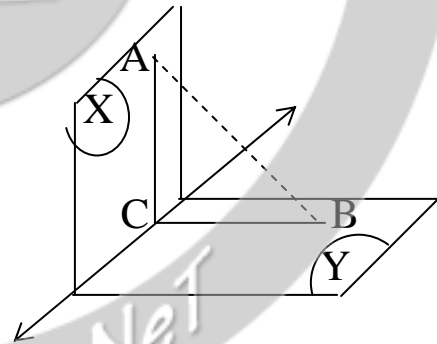
\overleftrightarrow{AB} and \overleftrightarrow{CD} are skew lines
 Draw $\overleftrightarrow{BN} \parallel \overleftrightarrow{CD}$ & $\overleftrightarrow{CM} \parallel \overleftrightarrow{BA}$
 Plane $ABN \parallel$ plane MCD



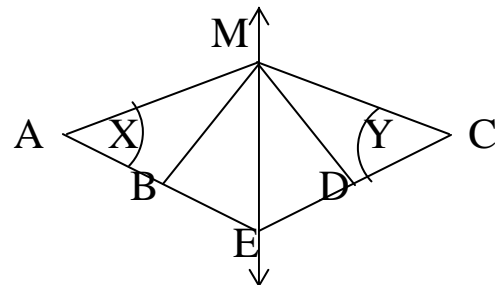
Examples

1) In the figure . Complete

- a) plane $ABC \cap X =$
- b) plane $ABC \cap Y =$
- c) plane $ABC \cap L =$
- d) plane $ABC \cap X \cap Y =$



2) If A, B, C and D are four coplanar points, such that $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$
 M is a point not belonging to their plane then find the line of intersection of the two planes MAB and MDC



$E \in$ plane $X, E \in$ plane Y
 $E \in X \cap Y$, similarly $M \in X \cap Y$
 $\therefore X \cap Y = \overleftrightarrow{ME}$

3) A, B and C are three non-collinear points lying in one plane

D, E and F are three non-collinear points lying in another plane

If $\widehat{AC} \cap \widehat{DF} = \{x\}$, $\widehat{AB} \cap \widehat{DE} = \{y\}$ and $\widehat{BC} \cap \widehat{EF} = \{z\}$. Prove that $z \in \widehat{XY}$

$X \in \widehat{FD}$ $X \in \text{plane FED}$

$X \in \widehat{AC}$ $X \in \text{plane ABC}$

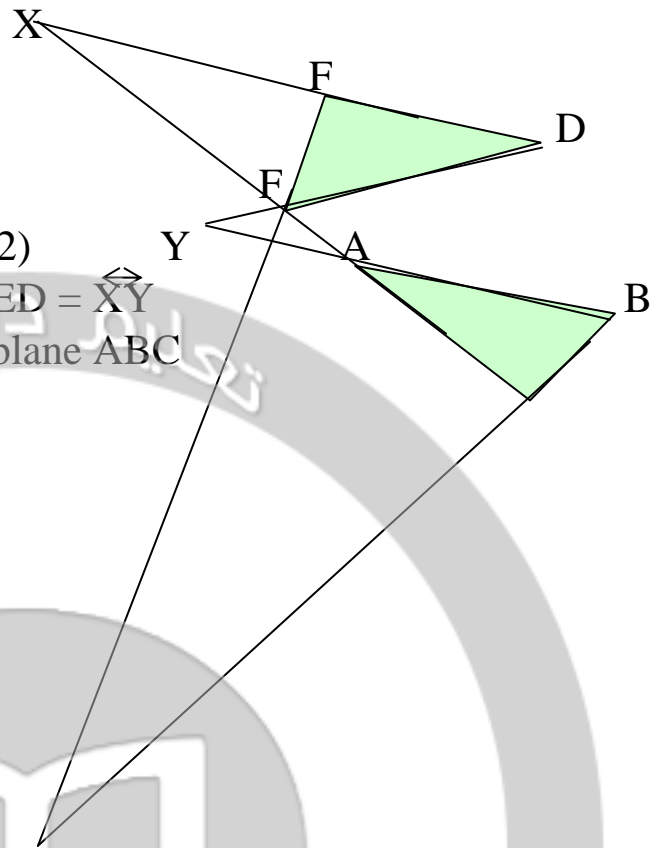
$\therefore X \in \text{plane ABC} \cap \text{EFD} \dots(1)$

Similarly $Y \in \text{plane EFD} \cap \text{ABC} \dots(2)$

From (1) & (2) $\text{plane ABC} \cap \text{plane FED} = \widehat{XY}$

$Z \in \widehat{EF}$ $Z \in \text{plane EFD} \ \& \ Z \in \text{plane ABC}$

$\therefore Z \in \widehat{XY}$



Geometrical fact :

If each of two lines is parallel to the third then they are themselves parallel .

EX: $\widehat{AB} \cap \text{plane X} = \{C\}$. \overline{AD} is drawn such that $\overline{AD} \cap \text{plane X} = \{D\}$

$E \in \overline{AD}$ and \overline{EB} intersect the plane X in O .

prove that C , O and D are collinear

A, B and D non collinear

then A , B and D form a plane Y

$C \in \overline{AB}$, $\overline{AB} \subset \text{plane Y}$

$C \in \text{plane Y}$, $D \in \text{plane X}$, $D \in \text{plane Y}$

$\text{Plane X} \cap \text{plane Y} = \widehat{CD}$

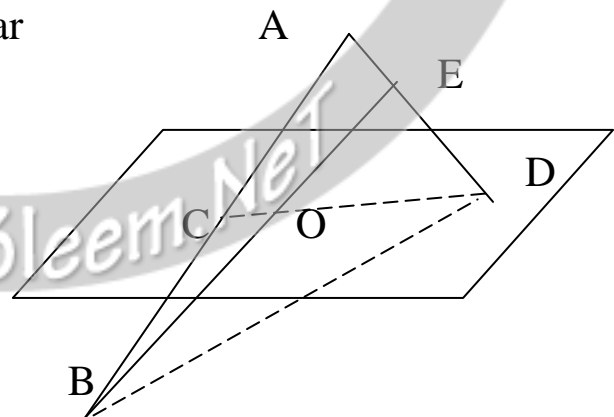
$O \in \overline{EB}$, $\overline{EB} \subset \text{plane Y}$

$O \in \text{plane Y}$, $O \in \text{plane X}$

O lies on the line of intersection of plane X and plane Y

$O \in \widehat{CD}$

O , C and D collinear



Exercise

Which of the following statements are true and which are false?

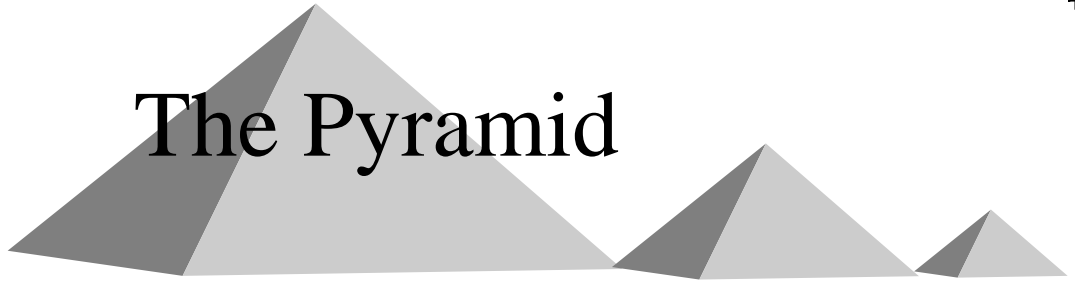
- 1) There is one and only one plane through three points in space
- 2) There is an infinite number of planes passing through two distinct points
- 3) There is one and only one plane passing through a point
- 4) There is an infinite number of planes passing through two intersecting lines
- 5) There is an infinite number of planes passing through two skew lines
- 6) There is one and only one plane passing through two parallel lines
- 7) There is one and only one plane passing through three non-collinear points
- 8) If a line $L \subset$ plane X , a point $A \in L$. So $A \in X$
- 9) Any two lines form a plane
- 10) Any two skew lines form a plane
- 11) Any two planes can intersected in 3 non – collinear points
- 12) Any two lines each of which is \parallel to the third are \parallel
- 13) Any two lines of which is \perp to the third are \parallel
- 14) If L_1, L_2 and L_3 are three lines in space, $L_1 \parallel L_2, L_1 \perp L_3$
Then L_1, L_2 and L_3 are coplanar
- 15) All vertical lines in space are \parallel
- 16) All horizontal planes are \parallel
- 17) All vertical planes are \parallel

- 18) If two lines do not intersect so they are //
- 19) If two planes have a line and a point outside it in common . then these two planes are intersecting
- 20) The planes containing two skew lines can be //
- 21) The planes containing two skew lines must be intersected
- 22) The sides of any quadrilateral are coplanar
- 23) If $\overleftrightarrow{AB} \cap \text{plane } X \neq \emptyset$ so A and B are in opposite sides of the plane X
- 24) if A and B are in the same side of plane X , then $AB \cap \text{plane } X = \emptyset$



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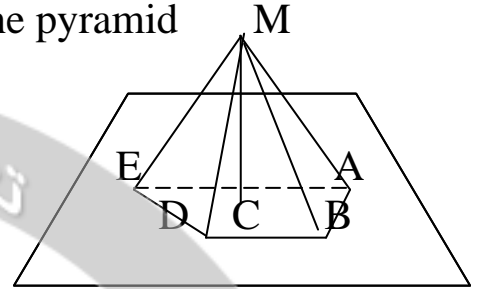
The Pyramid



In The figure

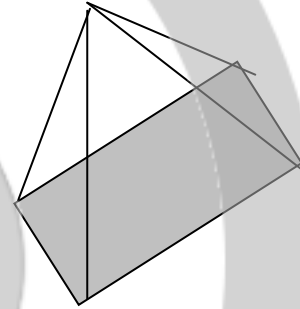
ABCDE is a polygon \subset plane X and M \notin plane X

The union of all segments \overline{MA} , \overline{MB} , Is called the pyramid



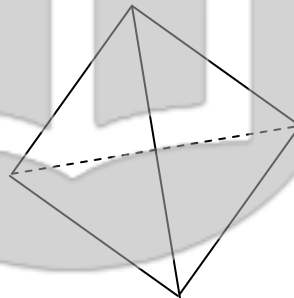
The type of pyramid according to the type of its base

The right pyramid. Whose its base is a regular polygon
And its lateral edges are equal



The regular pyramid (triangular)

All faces are equilateral triangles



Parallelism of a lines and a plane

Theorem (1)

If a line is parallel to a plane , then it is parallel to a line of intersection of this plane with the plane containing the given line .

Given : $\overleftrightarrow{AB} \parallel \text{plane Y}$ and $\overleftrightarrow{AB} \subset \text{plane X}$, $\text{plane X} \cap \text{plane Y} = \overleftrightarrow{CD}$

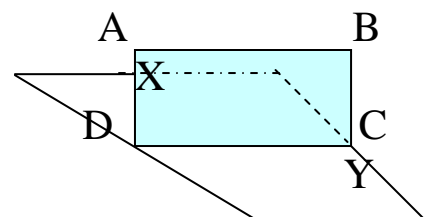
R.T.P : $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$

Proof: $\overleftrightarrow{AB} \parallel \text{plane Y}$ so $\overleftrightarrow{AB} \cap \text{plane Y} = \phi$, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \phi \dots (1)$

$\overleftrightarrow{AB} \subset \text{plane X}$, $\overleftrightarrow{DC} \subset \text{plane Y}$

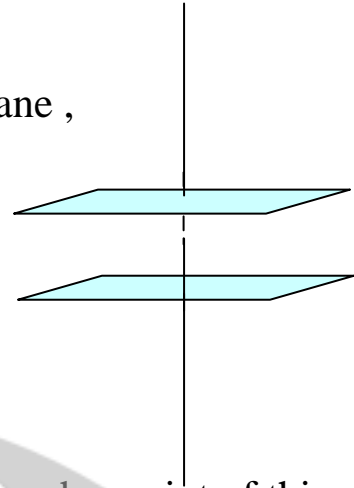
Then \overleftrightarrow{AB} and \overleftrightarrow{CD} are coplanar....(2)

Then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



Fact

If a line outside a plane is parallel to a line in the plane ,
then it is parallel to the plane



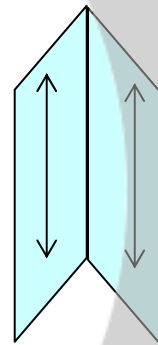
Corollaries

1) If a line intersects one of two parallel planes
then it intersects the other

2) If a line is // to a plane , then any line passing through a point of this plane
and // to the given line lies in the plane

3) If a plane intersects two parallel plane , then the lines of intersection are //

4) If two intersecting planes pass through two // lines
Then their line of intersection is // to those two lines



5) If a line is // to each of intersecting planes so it // to
It // to their line of intersection