Dynamics

* The Relative velocity

Velocity vector

Is a vector in which

1) Its magnitude is the value of velocity (speed)
2) Its direction is the direction of motion .

Units of speed :

Unit of distance / unit of time like cm/s , m/s , km /h , ……

Uniform velocity

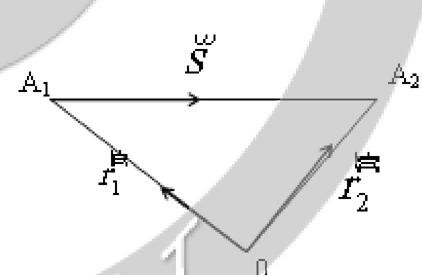
In which the magnitude and direction of motion are constants .

* The Relation between displacement vector and velocity vector

$$\vec{r}_1 + \vec{s} = \vec{r}_2$$

$$\vec{s} = \vec{r}_2 - \vec{r}_1$$

$$\vec{V}a = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\vec{s}}{t_2 - t_1}$$



if $t_2 - t_1 \neq o$, t is the middle of this time interval

$$\therefore \vec{V}_a = \frac{\vec{S}}{t}$$

Variable motion In which both the magnitude and direction of velocity or one of them is changed .

Relative velocity

$$\vec{V}_{ba} = \vec{V}_b - \vec{V}_a$$

Example 1

A cyclist covered 60 km on a straight way with a velocity of 20km/h , them he retunned back 30 km with a velocity of 15 km/h . find the average velocity during his whole trip .

Sol.

$$t_1 = \frac{60}{20} = 3h$$

$$t_2 = \frac{30}{15} = 2h$$

$$V_a = \frac{S}{t} = \frac{60-30}{3+2} = \frac{30}{5} = 6km/h$$

Example 2

A controlling speed car (radar) moves on a road at 40 km/h . it watched a truck coming from the other opposite road which seemed to be moving at 130 km/h find the real velocity of the truck

Sol.

$$\vec{V}_{ba} = \vec{V}_b - \vec{V}_a$$

$$130\hat{e} = \vec{V}_b + 40\hat{e}$$

$$\therefore \vec{V}_b = 90\hat{e}$$

$\therefore V_b =$ velcity of the truck = 90km/h.

* Umiform variable motion in a straight line

Def

The Acceleration vector ($\vec{A}$)

Is the time rate of velocity vector W.r.t . the time

$$\therefore \vec{A} = \frac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1}$$

Units of acceleration :

Unit of distance / (unit of time $)^2$

Like m/s^2 , cm / s^2 , ….

: Note

$\frac{5}{18} m/s$ = km/h1

$\frac{250}{9} cm/s$ = km/h 1

Rules of uniform variable motion *

v = u + At (1

^{2}At $\frac{1}{2}$ + S = ut (2

AS , where u is the initial velocity2 + ^{2}u = ^{2}V (3

V is the final velocity

, S is the distance

A is the acceleration

Example 1

A particle moved in a constant direction , it covened 18 m it seconds of its motion , 12 m during the 5th 3 during the 1st that the particle was second . prove second , 20 m during the 9th moving with uniform acceleration . calculate its velocity at the . begining of motion

.Sol

from t = o to t = 3 (1

the middle of the time interval = 1.5

$$V_{1.5} = \frac{18}{3} = 6 m/s \rightarrow (1)$$

from t = 4 to t = 5 (2

the middle of the time interval = 4.5

$$V_{4.5} = \frac{12}{1} = 12 m/s \rightarrow (2)$$

from t = 8 to t = 9 (3

$$V_{8.5} = \frac{20}{1} = 20 m/s \rightarrow (3)$$

$$A = \frac{v_2 - v_1}{t_2 - t_1} = \frac{12-6}{4.5-1.5} = 2 m/s^2 \rightarrow (4)$$

. from 1 , 2

$$A = \frac{V_2 - V_1}{t_2 - t_1} = \frac{20-12}{8.5-4.5} = 2 m/s^2 \rightarrow (5)$$

. from 2 , 3

(from (4) , (5

∴ the particle moving with uniform acceleration

$V = u + A\,t$

$V_{1.5} = u + 2t \Rightarrow 6 = u + 2 \times 1.5 \Rightarrow u = 3$ m/s

Example 2

A bullet is fired horizontally towards a wooden target with a velocity 200 m/s. it was penetrated 4 cm. find the uniform acceleration with which the bullet moved. if the target is 3 cm thickness, what is the velocity by which the bullet comes out.

Sol.

1) $u = 200$ m/s

$s = 0.04$ m

$v = 0$

$V^2 = u^2 + 2AS$

$0 = 40000 + 2 \times 0.04 \times A$

∴ $A = -500000$ m/s^2

2) $V^2 = u^2 + 2\,AS$

$V^2 = (200)^2 - 2 \times 500000 \times 0.03$

$V^2 = 10000$

∴ $V = 100$ m/s

* Vertical motion Under Gravity

* Laws of vertical Motion

1) $v = u + gt$

2) $S = ut + \frac{1}{2} gt^2$

3) $V^2 = u^2 + 2gs$

Where "g" is the acceleration of gravity , acts downwards

$g = 9.8$ m/s^2

or $g = 980$ cm/s^2

Notes :

(1 t = time of Max height = $\frac{u}{g}$

(2 S = Max height = $\frac{u^2}{2g}$

Remarks :

(1 the time reach max height = the time to return back to the point of projection

(2 The velocity of a body moving upwards when passes by a certain point equal in magnitude to the velocity of the body when moving downwards when passes by the same point .

Example 1

A particle is projected upwards with velocity 39.2 m/s . , find the time taken to reach max height and also the max height .

Sol.

(1 t = $\frac{u}{g} = \frac{39.2}{9.8} = 4 \text{ sec}$

(2 h = $\frac{u^2}{2g} = \frac{39.2 \times 39.}{2 \times 9.8} = 78.4\text{m}$

Example 2

A small ball is projected vertically upwards from the window of a house ,it was observed moving downwards against the window after 6 sec then reached the ground after 7 sec from the instant of projection . find the height of the window .

Sol.

Time to reach max height = $\frac{6}{2} = 3 \text{ sec}$

$$t = \frac{u}{g}$$

$$3 = \frac{u}{9.8}$$

$$\therefore u = 3 \times 9.8 = 29.4\text{m} / \text{s}$$

the time from the window to the ground ∴

sec $1 = 6 - 7 =$

$^2 g\, t \frac{1}{2} + S = ut$

$^2(1) \times 9.8 \times \frac{1}{2} + 1 \times S = 29.4$

S = 29.4 + 4.9

S = 34.3 m

* Differentiation of Vector functions

$\vec{r}$) t) is the position vector
$\vec{V}$) t) is the velocity vector
$\vec{A}$) t) is the Acceleration vector
$\vec{S}$) t)is the displacement vector

Rules :

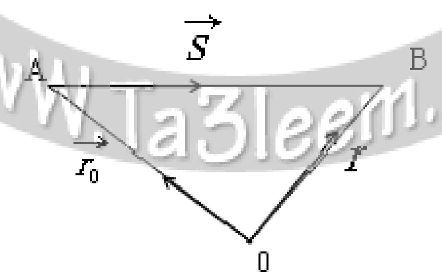
$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{c}$$

$$\vec{A} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{c}$$

where $\hat{c}$ is a unit vector

*The Relation between displacement vector and velocity vector

In fig



$\vec{r}_o + \vec{s} = \vec{r}$

$\therefore \vec{s} = \vec{r} - \vec{r}_o$

$$\frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt} - \frac{d\vec{r}_o}{dt}$$

$$\therefore \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt} - \vec{o}$$

$$\vec{V} = \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt}$$

Notes

(1if VA > o

the motion is accelerated

(2if VA < o

the motion is retarded

Types of Motion

(1uniform if A =o

(2uniformly changing

if A = constant ¹ o

(3Variable motion

If A is a variable

Example 1

Find the velocity and accalcretion vectors in the following cases , showing which is uniform , uniformly changing or variable motion

(1 $\vec{r} = (2t + 1)\hat{c}$

(2 $\vec{r} = (4t^2 + 5t - 2)\hat{c}$

(3 $\vec{r} = (2t^3 - t + 5)\hat{c}$

sol.

(1 $\vec{r} = (2t + 1)\hat{c}$

$$\vec{V} = \frac{d\vec{r}}{dt} = 2\hat{c}$$

$$\vec{A} = \frac{d\vec{V}}{dt} = o\vec{c}$$

the motion is uniform

2) $\vec{r} = (4t^2 + 5t - 2)\hat{c}$

$$\vec{V} = \frac{d\vec{r}}{dt} = (8t + 5)\hat{c}$$

$$\vec{A} = \frac{d\vec{V}}{dt} = 8\hat{c}$$

The motion is uniformly changing

3) $\vec{r} = (2t^3 - t + 5)\hat{c}$

$$\vec{V} = \frac{d\vec{r}}{dt} = (6t^2 - 1)\hat{c}$$

$$\vec{A} = \frac{d\vec{v}}{dt} = 12t\hat{c}$$

the motion is variable

Example 2

If $\vec{S} = (2t^3 - t)\hat{c}$, find the velocity and acceleration vectors and show when will the motion be retarded and when it will be accelerated .

Sol .

$$\vec{S} = (2t^3 - t)\hat{c}$$

$$\vec{V} = \frac{d\vec{s}}{dt}$$

$$\therefore \vec{V} = (6t^2 - 1)\hat{c}$$

$$\vec{A} = \frac{d\vec{v}}{dt}$$

$$\therefore \vec{A} = 12t \quad \hat{c}$$

$$VA = (6t^2 - 1)(12t)$$

$$\because t > o$$

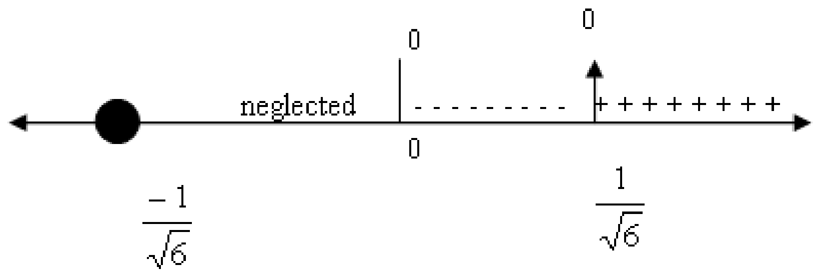
$$\therefore 6t^2 - 1 = o$$

$$\text{or} \quad 6t^2 - 1 < o$$

$$\text{or} \quad 6t^2 - 1 > o$$

$$\text{when} \quad 6t^2 - 1 = o$$

$$\therefore t = \mp \frac{1}{\sqrt{6}}$$



when t < $\frac{1}{\sqrt{6}}$
the motion is accelerated

when o < t > $\frac{1}{\sqrt{6}}$
The motion is retarded

* Newton's laws for motion.

(* Def : The Momentum . ($\vec{H}$

. The momentum of a body is the product of its mass times its velocity

$\therefore \vec{H} = m\vec{v}$

: units of mass

.. , like gm, k.gm , ton
kgm ^{3}ton = 10 1
gm ^{3}kgm = 10 1
m.gm ^{3}gm = 101

Units of momentum

H = m v $\because$

.unit of momentum = unit of mass. Unit of velocity $\therefore$

... ; like gm.cm/s , kgm . km/h

($\Delta\vec{H}$) **Change in momentum**

($\vec{V}_2 - \vec{V}_1$) m = $\Delta\vec{H}$

Example 1

Find the momentum of a car of mass 1600 kgm moving with velocity 90 km/h giving your answer in gm.cm/sec

.Sol

$H = mv$

kgn. Km./h $\quad 90 \times H = 1600$

kgm.km/h $\quad 144000 =$

$10^3 \times 144 =$

gm.cm/s $\frac{250}{9} \times 10^3 \times 10^3 \times 144 =$

gm $10^9 \times 4 =$

Example 2

A rubber ball of mass 100 gm is let to fall from a height of 19.6 m. on a horizontal ground , it impinges with it and then rebounded to a height 10 m before it comes to instantenous rest calculate the magnitude of the change of its momentum just before and after the impact

.Sol

$gs2 + {}^2u = {}^2V$

$19.6 \times 9.8 \times o + 2 = {}^2V$

$V = 19.6$ m/s

$\frac{u^2}{2g} = h$

$\therefore 10 = \frac{u^2}{2 \times 9.8}$

$u = 14$ m/s

$\Delta \vec{H} = m(\vec{V}_2 - \vec{V}_1)$

$\Delta H = (0.1)(14 + 19.6) \therefore$

$= 3.36$ kgm.m/s

* Newton's 1st law

" Every body preserves in its state of rest or moving uniformly except in so far it is mode to change that state by external effect"

l.e. in these 2 cases.

$F = R$

Example(1)

A particle of mass m moves under the effect of 2 forces
$\vec{F}_2 = -m\hat{j}$ find the additional force which if acts on , $\vec{F}_1 = 4m\hat{i}$
.the particle it will move uniformly

.Sol

$\vec{F} = \vec{f}_1 + \vec{f}_2$

$\vec{f} = 4m\hat{i} - 5m\hat{j}$

$\therefore$ The addtional froce = $-4m\hat{i} + 5m\hat{j}$

(Example(2

A metal ball of weight 80 Kgm.wt descends vertically in a liquid , it is found. That it travels equal distances in Equal time .intervals. Find the resistance of the liquid

.SOL

The motion is uniform $\because$

$\therefore$ f = R

$\therefore$ R = w = m g

$\therefore$ .R = 80 Kgm. Wt

(Example(3

A truck of mass 6 tons moves on a straight horizontal road under the action of a resistance directly proportional to the magnitude of the velocity. If the resistance is 10 Kgm. Wt per ton of the mass when the velocity is 64 Km/h . Find the max velocity given that the max force generated by the .motor is 90 Kgm.wt

.Sol

When the motion is uniform

$\therefore\ f = R$

$\because\ R \propto V$

$\frac{R_1}{R_2} = \frac{V_1}{V_2}$

$\therefore\ \frac{10x6}{90} = \frac{64}{V_2}$

$V_2 = \frac{64x90}{10x6} = 96Km / h.$

law s 2^{nd}'Newton *

Rate of change of momentum w.r.t. to the time is propartional to the " impressed force and takes place in the direction in which the force is "impressed

Derivation of the law

$\frac{d}{dt}(mv) \propto F$

iF m is constant

$\therefore\ m \frac{dv}{dt} \propto f$

$\therefore\ m\ A \propto f$

$\therefore\ m\ A = K\ F$

K=1 ⇒ f=1 dyse , ^{2}if m=1 gm, A=1 Cm/s

K=1 ⇒ F =1 Newton , ^{2}If m=1 Kgm, A=1m/s

F = m A

.In the case of the existence of resistance, we can write

F - R = m A

Notes

dyne ^{5}N = 101 (1)

. gm.wt = 980 dyne1 (2)

.Kgm.wt = 9.8 Newton1 (3)

(Example(1

Aparticle of mass m moves under the effect of 2 forces

$\vec{F}_1 = 2m\hat{i} - 5m\hat{j}. , \vec{F}_2 = m\hat{i} + m\hat{j}$

Find the acceleration vector and find its magnitude

Sol.

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\therefore \vec{F} = 3m\hat{i} - 4m\hat{j}$$

$$\vec{f} = m(3\hat{i} - 4\hat{j})$$

$$\therefore \vec{A} = 3\hat{i} - 4\hat{i}$$

$$\|\vec{A}\| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = 5$$

Example(2)

A mass 2 Kgm. Falls 10m to rest by peretrating 5 Cm into some sand . find the Resistance of sand supposing it is uniform.

Sol

$$V^2 = u^2 + 2 g s$$

$$\therefore V^2 = o + 2 \times 9.8 \times 10$$

$$\therefore V = 14 \text{ m/s}.$$

in side the sand.

$$V^2 = u^2 + 2 a s$$

$$\therefore o = (14)^2 + 2 \times 0.05 a$$

$$\therefore a = -1960 \text{ m/s}^2$$

$$\therefore f - R = m a$$

$$2 \times 9.8 - R = 2 \times -1960$$

$$\therefore R = (2 \times 9.8 + 2 \times 1960) \text{ N}$$

$$\therefore R = 402 \text{ Kgm.wt}.$$

Example (3)

A metal ball of mass m gm. Moves along a straight line with a uniform velocity 15m/s. in a dusty medium . If the rate of accumlation of dust to the surface of the ball is 0.04 gm/Sec.

Find the force acting on the ball .

.Sol

$$F = \frac{d}{dt}(mv)$$

$$\therefore F = V\frac{dm}{dt}$$

$$F = (1500)(0.04)$$

$$\therefore F = 60 dyne.$$

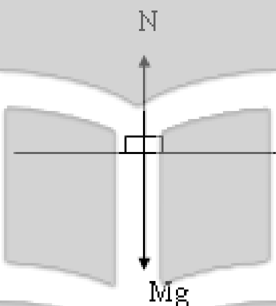
law s 3rd'Newton *

to every action thene is a reaction equal in magntude and opposite in “
.”direction

Applications

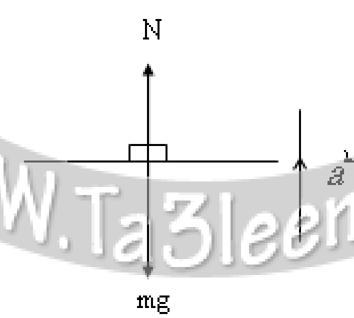
.A body placed on the floor of a lift moving with simple acceleration :st1

.i) If the lift at rest or moving uniformly



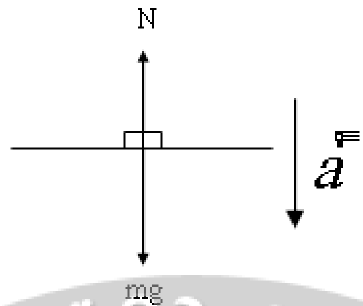
N = w
N = mg

.ii) Moving up wards



N > mg
(N = m (g + a

.iii) Moving down wards

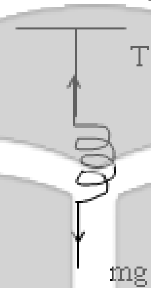


$N < mg$

$(N = m (g - a$

2nd : Abody suspended by aspring balance from the top of a lift.

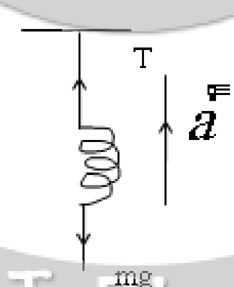
i) at rest or moving uniformly.



$T = w$

$T = m g$

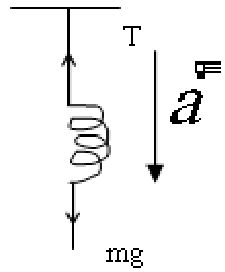
ii) Moving up wards.



$T > m g$

$(T = m (g + a$

iii) Moving down wards.

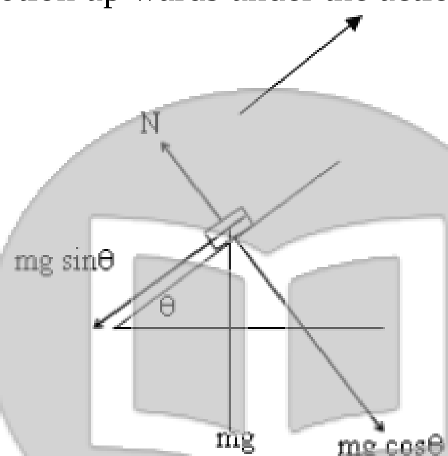


T < m g

(T = m (g - a

.The motion of a body on a smooth inclined plane : rd3

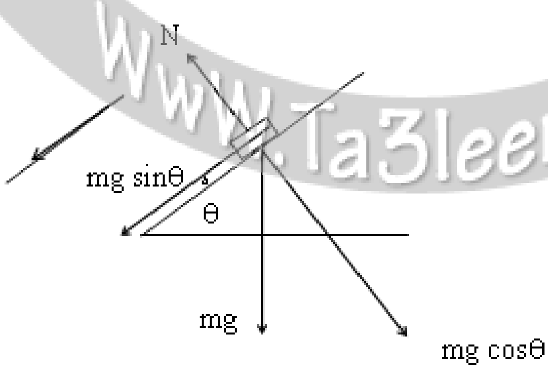
.i) if the motion up wards under the action of the weight only



θ N = m g cos

θ a = - g sin

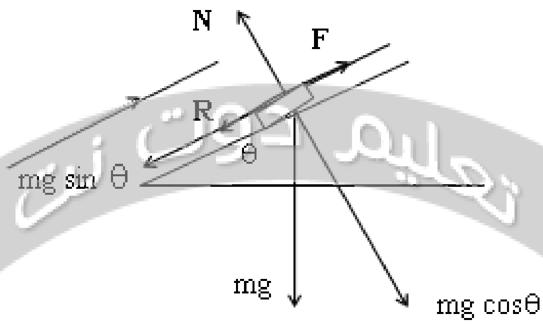
.ii) If the motion down wards under the action of the weight only



θ N = m g cos

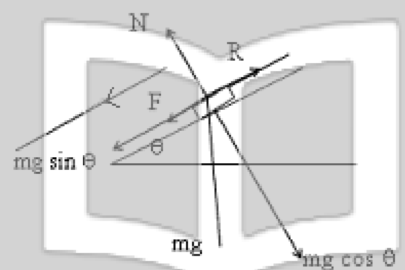
θ a = g sin

.iii) If there is a force and resistance , the motion upwards



m A = θ F-R -mg sin

.iv) if there is a force and resistance , the motion down wards



F - R = m a + mg sinθ

(Example (1

Abody of mass 4 Kgm. Is placed on the floor of a lift . find the .pressure force of this body on the floor of the lift when the lift

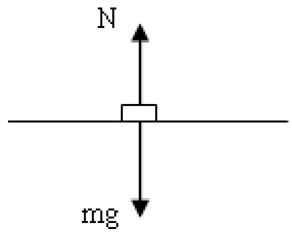
.is moving with uniform velocity(i

.^{2}is moving upwards with are = 147 Cm/s(ii

.^{2}is moving downwards with are = 147 Cm/s(iii

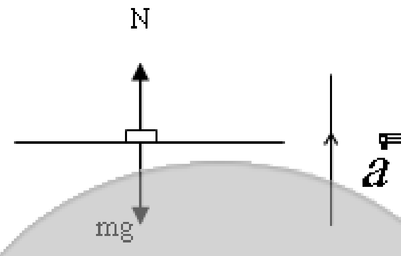
SOL

N = m g(i



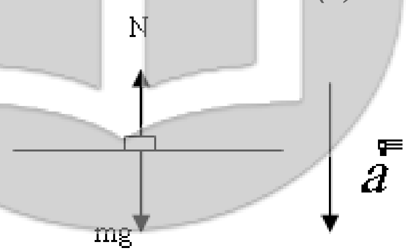
N = 4 x 9.8 N
.N = 4 Kgm.wt

(b)N = m (g + a



N = 4 (9.8 + 1.47) N
.N = 4.6 Kgm. Wt

(c)N = (g - a

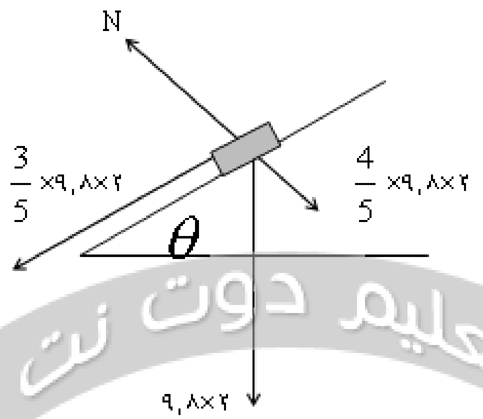


N = 4 (9.8-1.47)N
.N = 3.4 Kgm.wt

(Example(2

Abody of mass 2Kgm is placed on a smooth plane inclined to then $\theta = \frac{3}{4}$ where tan ,θ° the horizontal of an angle of measure is left to move find the magnitude of the force of reaction of the plane on the body , find also the magnitude of the acceleration .on the plane

.Sol



$\because \tan\theta = \frac{3}{4}$

$\therefore \sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$N = m\,g\cos\theta$

$= 2x9.8x\frac{4}{5}\,N$

$= 1.6Kgm.wt$

$a = 9.8x\frac{3}{5} = 5.88m/s^2$

Impulse and collision *

Def: Impulse $(\bar{I} = \bar{f}t)$

The impulsive vector is the product of the force vector times the time.

$\therefore I = f\,t$

Units of Impulse

Unit of force . unit of time life dayn .sec, N.sec,

Theorem

Rate of change of momentum is equal to the impulse

Proof

$$\vec{H} = m\vec{v}$$

$$\frac{dH}{dt} = \lim_{t \to 0} \frac{H' - H}{t} \cong \frac{H' - H}{t}$$

$$\because \frac{dH}{dt} = f$$

$$\therefore \frac{H' - H}{t} = f$$

$$\therefore H' - H = ft$$

$$\therefore I = m(V' - V)$$

*collision of 2 smooth sphers **

If 2 smooth sphers collide

∴ The impulse of the 1^{st} sphere on the 2^{nd} sphere equal in magnitude and opposite in direction to the impulse of the 2^{nd} sphere on the 1^{st} sphere.

$$\therefore M_1 (V_1' - V_1) = -m_2 (V_2' - V_2)$$

$$\therefore M_1 V_1 + M_2 V_2 = M_1 V_1' + M_2 V_2'$$

Example (1)

A rail road car of mass 21 tons moves with velocity 14m/s , A barrier of collision brought it to rest through 0.3 second. Find the magnitude of the impulse and the magnitude of the average force.

Sol.

$$I = m(V` - V)$$
$$= 21x10^3(0-14)$$
$$= -294x10^3 \quad N\text{-}sec$$
$$I = Ft$$
$$\therefore -294x10^3 = -F x 0.3$$
$$\therefore F = \frac{294x10^3}{0.3} = 9.8x10^5 N$$
$$\therefore F = 10^5 \text{ hgm.wt} = 100\text{ton.wt}$$

(Example(2

Smooth sphers of masses 0.1 Kgm, 0.2 Kgm are moving on a 2 horizontal ground in the same straight line . the velocity of the is 2 m/s in an opposite is 1m/s and the velocity of 2nd 1st spere if the 2 sphers collide so that the 2nd . direction of the 1st moves in the same direction with velocity 0.75 m/s after sphere and the impulse of the impact, find the velocity of the 1st .sphere on it 2nd

Sol

$$M_1V_1 + M_2V_2 = M_1V_1^{\backslash} + M2V_2^{\backslash}$$
$$0.1x1 + 0.2x-2 = 0.1xV_1^{\backslash} + 0.2x-0.75$$
$$\therefore \frac{0.1-0.4+0.2x-.75}{0.1} = V`$$
$$\therefore V` = -1.5m/s$$

.sphere moves with velocity 1.5 in its opposite direction The 1st ∴

(I = m (V` − V

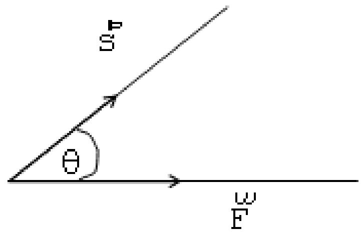
(1+ 1.5) 0.1 =

.N.Sec 0.25 =

Work - power - Energy

* Work

Def: The work is scalar product of the force times the displacement vectors.



$w = \vec{f} . \vec{S}$

$w = f \, s \cos\theta$

and if $\vec{F}$ in the direction of $\vec{S}$

$\therefore \; w = f \, S$

units of work

unit of force . unit of distance like .
dyne cm = erg, - Newton meter = joule
gm wt.cm , …
1 joule = 10^7 erg

Example(1)

A particle moves in a straight line from the point A(-1.4) to the point B(2,-1) under the action of a force $\vec{f}$ = -2î-3j., Calculate the work done by this force.

Sol.

$$\vec{S} = \vec{AB} = \vec{B} - \vec{A} = (2,-1) - (-1,4) = (3,-5)$$

$$w = \vec{F} \Theta \vec{S} = (-2,-3) \Theta (3,-5) = -6 + 15$$

$$\therefore w = 9 \;\; \text{units of work.}$$

Example(2)

A man ascends a straight road inclined at an angle of measure =θ° to the horizontal . he moved a distance = ℓ then he returned to the strating point find the work done by its weight through the whole journey.

Sol.

$$\vec{S} = \vec{o}$$

$$\therefore w = \vec{F} \Theta \vec{S} = O$$

Example(3)

Find the work done by the weight when a body of mass 3 tons is raised up wards a distance 9 meters.

Sol.

$$w = \vec{F} \Theta \vec{S}$$

$$w = (-mg)(S)$$

$$= -3x10^3 x9.8x9$$

$$= -264600 \quad \text{Jaule}$$

* Power

Def: the power is the time rate of doing work .

$$\therefore \text{Power} = \frac{dw}{dt} = \frac{d}{dt}(fs) = F\frac{ds}{dt} = Fv$$

$$\therefore \text{Power} = FV$$

Units of power

Unit of force . unit of velocity Kgm. Wt. M/s, gm.wt. Km/h, N.m/s - watt.

1 Kilo watt = 10^3 watt

1 Kgm. Wt.m/s = 9.8 watt

1 horse = 75 Kgm. Wt. M/s.

$\therefore$ 1 horse = 735 watt

Note

To calculate the power , the velocity must be maximum.

Example(1)

If the displacement vector as a function of the time of a particle of unit mass is $\vec{S} = 7ti + (3t-1)\hat{j}$, and if the force acting on this particle is $\vec{F} = 5\hat{i} - 3\hat{j}$ find the power of this force.

Sol.

$$\frac{d}{dt}(\vec{F}\Theta\vec{S})$$

$$\text{power} = \frac{d}{dt}[(7t, 3t-1).(5,-3)]$$

$$= \frac{d}{dt}[35t - 9t + 3]$$

$$= 35 - 9 = 26 \quad \text{unit of power.}$$

Example(2)

A train of mass 250 tons is moving on a horizontal rails with uniform velocity 30 Km/h., find the power of the engine of the train , gwen that the road resistance is equal to 9 Kgm.wt per ton of its mass.

Sol.

$\because$ The velocity is uniform.

$\therefore$ F =R

$$\because R = 9x9.8x250 \quad N$$

$$\therefore F = 9x9.8x250 \quad N$$

$$v = 30Km/h = 30x\frac{5}{18} \quad m/s$$

$$\text{Power} = F\,v$$

$$\therefore \text{power} = 9 \text{ x } 9.8 \text{ x } 250 \text{ x } 30 \text{ x } \frac{5}{18}$$

$$\therefore \text{power} = 183750 \quad \text{watt}$$

$$\therefore \text{power} = 250 \quad \text{horses}$$

* Kinetic Energy(T)

Def: is defined as holf the product of the mass times the square of velocity

$$T = \frac{1}{2}mv^2.$$

Units of "T" : The same unit of work.

i)if m in gm, V in cm/s $\Rightarrow$ T in erg

ii) if m in Kgm, V in m/s $\Rightarrow$ T in joule.

Note : Change in T = $\frac{1}{2}m(V_2^2 - V_1^2)$

Example(1)

Find the kinetic Energy of a body of mass 40 gm. Moving with velocity 30 m/s.

Sol.

$$T = \frac{1}{2} mv^2$$

$$= \frac{1}{2} x40x(3000)^2$$

$$= 18x10^7 erg = 18 Joule$$

Example(2)

A body is moving with velocity $\vec{v} = 30\hat{i} + 40\hat{j}$, the magnitude of velocity being measured in units of cm/s. find the mass of this body if its kinetic energy is equal to 2.5 joules.

Sol.

$$v = \sqrt{(30)^2 + (40)^2} = 50 Cm/s. = 0.5 m/s$$

$$T = \frac{1}{2} m v^2$$

$$\therefore 2.5 = \frac{1}{2} m(0.5)^2 \qquad (X2)$$

$$\therefore 5 = m \times 0.25$$

$$\therefore m = \frac{5}{0.25} = 20 Kgm.$$

* Principle of work and Energy

$$T = \frac{1}{2} mv^2$$

$$\frac{dT}{dt} = \frac{1}{2} m\left(2v\frac{dv}{dt}\right)$$

$$= mvA$$

$$\therefore \frac{dT}{dt} = FV = Power \rightarrow (1)$$

$$\frac{dw}{dt} = FV = Power \rightarrow (2)$$

at t = o

$\therefore T_o - O = constant$

$\therefore T - w = T_o$

$\therefore T - T_o = w$

Use the principle of work and Energy to solve the following problems:-

Example(1)

A body of mass 3 kgm. Is let to fall from a height 20 meters above the earth's surface. Find the kinetic energy when it is about to hit the ground.

Sol.

$T - T_o = w$

$T - o = m\,g\,s$

$T = 3 \times 9.8 \times 20$

$T = 588$ Joules

Example(2)

A bullet is fired horizontally with velocity 350 m/s at a piece of wood, it is embedded in it at a depth of 4 Cm. If a similar bullet is fired with the same velocity at a fixed target made of the same wood of thickness 3 Cm, what is the velocity with which the bullet comes out of the target assuming that the resistance constant.

Sol.

$T - T_o = w$ (i

$o - \frac{1}{2} m \times (350)^2 = w$

$\therefore -\frac{1}{2} m \times (350)^2 = R \times 0.04$

$\therefore P = \dfrac{-\frac{1}{2} mx(350)^2}{0.04}$ N

$T - To = w$ (ii

$T - \frac{1}{2} m \times (350)^2 = R \times 0.03$

$\dfrac{-\frac{1}{2} mx(350)^2 x0.03}{0.04} \times T - \frac{1}{2} m \times (350)^2$

$$\therefore T = \frac{1}{2} m \times (350)^2 = (1- \tfrac{3}{4})$$

$$\therefore T = \frac{1}{2} m \times (350)^2 \times \tfrac{1}{4}$$

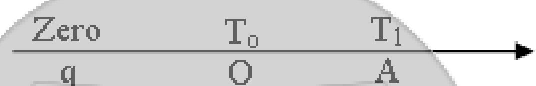
$$T = \frac{1}{2} m \times \left(\frac{350}{2}\right)^2$$

$$\therefore V = \frac{350}{2} = 175 m/s.$$

* Potential Energy (ρ(

Def: The potential energy of a particle at a certain point (instant) defined by " the work done by the acting force on the body if it is moved it from its position at this point to a fixed point on the straight line on which the motion occurs".

In Fig.



$$P_A = \vec{F} \ominus A\vec{q} \rightarrow (1)$$

$$P_o = \vec{F} \ominus O\vec{q} \rightarrow (2)$$

$$From (2) - (1)$$

$$\therefore P_A - P_o = \vec{F} \ominus A\vec{q} - \vec{F} \ominus O\vec{q}$$

$$\therefore P_A - P_o = -w$$

$$\because T - T_o = w$$

$$\therefore p - p_o = -(T - To)$$

$$\therefore P + T = P_o + T_o$$

∴ The sum of potential Energy and kinetic Energy is constant .

Units of P.E. : The same units of work,

Notes:−

(1) In the case of vertical motion

P = m g h(i

Change in P.E = DP = mg $(h_2 - h_1)$(ii

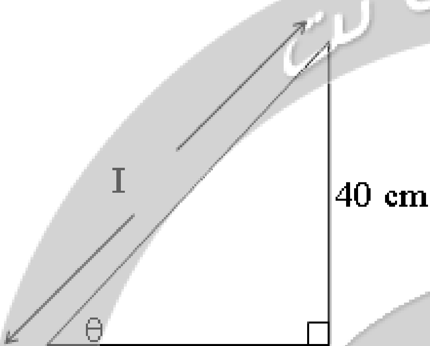
(2) in the case of inclined plane

p= m g I sin q

Example(1)

Find the P.E. of a body of mass 6 Kgm. At a height of 40 Cm above the surface of the earth, giving your answer in erg.

Sol.

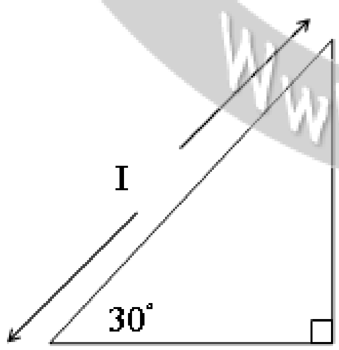


$$P.E = m\ g\ h$$
$$= 6 \times 10^{3} \times 980 \times 40$$
$$= 23.52 \times 10^{7} \text{ erg}$$

Example(2)

A particle of mass 50 gm descends a distance 40Cm along a line of greatest slope of a a smooth inclined plane whose inclination to the horizontal is an angle of measure 30°. Find the change in its potential energy.

Sol.



$$\Delta p = m\ g\ I\ \sin\theta$$

$= 50 \times 980 \times 40 \times \frac{1}{2}$

$= 9.8 \times 10^4$ ergs.