

3rd year secondary

last look 2017

Mr./Mohamed Ali

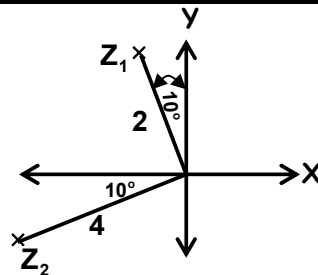
Algebra and solid geometry

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Q(1) In the opposite figure:

find the exponential form of $\frac{Z_1}{Z_2}$

$$\frac{2(\cos 100^\circ + i \sin 100^\circ)}{4(\cos -170^\circ + i \sin -170^\circ)} = \frac{1}{2}(\cos 270^\circ + i \sin 270^\circ) = -\frac{1}{2}i$$



Q(2) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1 Z_2) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 = \dots\dots$

$$\theta_1 + \theta_2 = \frac{5\pi}{18} \quad \text{and} \quad \theta_1 - \theta_2 = \frac{\pi}{9} \quad \therefore \theta_1 = \frac{7\pi}{36}$$

Q(3) 4 non collinear and coplanar points. Find the number of line segments joining each two of them?

$${}^4C_2 = 6$$

Q(4) $1 + 3\omega + 3\omega^2 = \dots\dots$

$$1 + 3(\omega + \omega^2) = 1 + 3 \times -1 = -2$$

Q(5) If ${}^{x+y}P_4 = 360$, $2X + Y = 5040$ then ${}^yC_{2x} = \dots\dots$

$$X + Y = 6, \quad 2X + Y = 7 \quad \therefore x = 1, y = 5 \quad {}^5C_2 = 10 \therefore$$

Q(6) If ${}^{n+2}P_r = 2 \times {}^{n+2}C_r$, $\frac{{}^n C_{r+1}}{{}^n C_r} = \frac{5}{3}$ find the value of ${}^{2n}C_{n-r} + {}^{n+3}P_{r-1}$

$$\therefore {}^{n+2}P_r = 2 \times {}^{n+2}C_r \quad \therefore {}^{n+2}P_r = 2 \times \frac{{}^{n+2}P_r}{r} \quad \therefore r = 2 \quad \therefore r = 2$$

$$\frac{{}^n C_3}{{}^n C_2} = \frac{n-3+1}{3} = \frac{5}{3} \quad \therefore n-2=5 \quad \therefore n=7$$

$${}^{2n}C_{n-r} + {}^{n+3}P_{r-1} = {}^{14}C_5 + {}^{10}P_1 = 2002 + 10 = 2012$$

Q(7) $\omega^2 \left(1 - \frac{1}{\omega^2} + \omega^2\right) = \dots\dots$

$$\omega^2(1 - \omega + \omega^2) = \omega^2 \times -2\omega = -2\omega^3 = -2$$

Q(8) Find the multiplicative inverse $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \\ 1 & 5 & 21 \end{pmatrix}$ $|A| = -1$

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} -3 & 1 \\ 5 & 21 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 21 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 5 & 21 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 21 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 5 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} -68 & -41 & 13 \\ 31 & 19 & -6 \\ 5 & 3 & -1 \end{pmatrix} \therefore A^{-1} = \frac{1}{|A|} \begin{pmatrix} -68 & 31 & 5 \\ -41 & 19 & 3 \\ 13 & -6 & -1 \end{pmatrix}$$

$$A^{-1} = -1 \begin{pmatrix} -68 & 31 & 5 \\ -41 & 19 & 3 \\ 13 & -6 & -1 \end{pmatrix} = \begin{pmatrix} 68 & -31 & -5 \\ 41 & -19 & -3 \\ -13 & 6 & 1 \end{pmatrix}$$

Q(9) Find $Z = \frac{-8}{1+\sqrt{3}i}$ where $i^2 = -1$ in the trigonometric form then find the two square roots of the number Z in the exponential form

$$Z = \frac{-8}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{-8(1+\sqrt{3}i)}{1-(\sqrt{3}i)^2} = \frac{-8(1-\sqrt{3}i)}{1+3} = -2(1-\sqrt{3}i) = -2+2\sqrt{3}i$$

$$X = -2, Y = 2\sqrt{3}, r = \sqrt{X^2 + Y^2} = \sqrt{2^2 + (2\sqrt{3})^2} = 4 \quad \tan\theta = \frac{Y}{X} = \sqrt{3} \quad (-,+)$$

θ in the 2nd $\therefore \theta = 180^\circ - 60^\circ = 120^\circ$

$$Z = 4(\cos 120^\circ + i \sin 120^\circ)$$

$$\sqrt{Z} = 2 \left(\cos \frac{120^\circ + 2K\pi}{2} + i \sin \frac{120^\circ + 2K\pi}{2} \right)$$

$$\text{When } K = 0 \therefore Z_1 = 2 \left(\cos \frac{120^\circ}{2} + i \sin \frac{120^\circ}{2} \right) = 2e^{\frac{\pi}{3}i}$$

$$\text{When } K = -1 \quad 2(\cos -120^\circ + i \sin -120^\circ)$$

Q(10) If the X axis cut the sphere which center (3,-4,12) and its radius length 13cm at the two points A and B then AB equals

$$(X-3)^2 + 16 + 144 = 13^2 \therefore (X-3)^2 = 9 \therefore X = 0 \text{ or } X = 6$$

$$\therefore \text{the two points } A(0,0,0) \text{ and } B(6,0,0) \therefore AB = 6$$

Q(11) The second , third and fourth terms in the expansion of $(X + a)^n$ according to the descending power of X are : 16,112,448 find the value of X , a , n

$$\frac{T_3}{T_2} = \frac{112}{16} = 7 \quad \therefore \frac{n-2+1}{2} \times \frac{a}{X} = 7 \quad \therefore \frac{n-1}{2} \times \frac{a}{X} = 7 \rightarrow (1)$$

$$\frac{T_4}{T_3} = \frac{448}{112} = 4 \quad \therefore \frac{n-3+1}{3} \times \frac{a}{X} = 4 \quad \therefore \frac{n-2}{3} \times \frac{a}{X} = 4 \rightarrow (2)$$

$$\frac{n-1}{2} \times \frac{3}{n-2} = \frac{7}{4} \quad \therefore 12(n-1) = 14(n-2) \quad \therefore 2n = 16 \quad \therefore n = 8$$

$$\frac{7}{2} \times \frac{a}{X} = 7 \quad \therefore \frac{a}{X} = 2 \quad \therefore a = 2X \quad , \quad T_2 = 16 = {}^8C_1 (a)(X)^7$$

$$16 = 8 \times 2X \times X^7 \quad \therefore X^8 = 1 \quad \therefore X = \pm 1$$

Q(12) prove that: $\begin{vmatrix} 1 & 1 & 1 \\ L & m & n \\ L^2 & m^2 & n^2 \end{vmatrix} = (L - m)(m - n)(n - L)$

$$\begin{aligned} & \begin{matrix} C_2 - C_1, C_3 - C_1 \\ \begin{vmatrix} 1 & 0 & 0 \\ L & m-L & n-L \\ L^2 & m^2-L^2 & n^2-L^2 \end{vmatrix} \end{matrix} = (m-L)(n-L) \begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 1 \\ L^2 & m+L & n+L \end{vmatrix} \\ & \begin{matrix} C_3 - C_2 \\ \therefore (m-L)(n-L) \begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ L^2 & m+L & n-m \end{vmatrix} \end{matrix} = (m-L)(n-L)(n-m) \end{aligned}$$

Q(13) How many ways we can put(distribute) 10 identical balls into 6 distinct bins

$${}^{6+10-1}C_{10} = 3003 \quad {}^{n+r-1}C_r \quad n = 6 \quad , \quad r = 10$$

Q(144) The least positive integer n which makes

$${}^{n-1}C_5 + {}^{n-1}C_6 < {}^nC_7 \quad \text{Is } \dots\dots\dots$$

$$\therefore {}^{n-1}C_5 + {}^{n-1}C_6 < {}^nC_7 \quad \therefore {}^nC_6 < {}^nC_7 \quad \therefore \frac{\lfloor n \rfloor}{\lfloor 6 \rfloor \lfloor n-6 \rfloor} < \frac{\lfloor n \rfloor}{\lfloor 7 \rfloor \lfloor n-7 \rfloor}$$

$$\therefore \frac{1}{n-6} < \frac{1}{7} \quad \therefore 7 < n-6 \quad \therefore n > 13 \quad \therefore n = 14$$

Q(15) If ${}^{n+1}P_{r+1} : {}^{n+1}C_{r+1} = 720$, ${}^nC_{r-2} + {}^nC_{r-3} = 56$ find the value of n , r

$${}^{n+1}P_{r+1} \div \frac{{}^{n+1}P_{r+1}}{r+1} = 720 \quad \therefore \underline{r+1} = 720 \quad \therefore r+1 = 6 \quad \therefore r = 5$$

$${}^nC_{r-2} + {}^nC_{r-3} = {}^nC_3 + {}^nC_2 = {}^{n+1}C_3 = 56 = {}^8C_3 \quad \therefore n+1 = 8 \quad \therefore n = 7$$

Q(16) In the expansion: $\left(X^2 - \frac{1}{X}\right)^9$ Find ① general term

② The term free of X

To get the General term:

$$T_{r+1} = {}^9C_r (-X^{-1})^r \times (X^2)^{9-r} = {}^9C_r (-1)^r X^{-r} \times X^{18-2r} = {}^9C_r (-1)^r \times X^{18-3r}$$

To get the term free of X

$$18 - 3r = 0 \quad \therefore 3r = 18 \quad \therefore r = 6 \quad \therefore T_7 = {}^9C_6 (-1)^6 \times X^{18-3 \times 6} = 84$$

Q(17) If ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 5 : 10 : 14$ find the values of n and r

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n - (r+1) + 1}{r+1} = \frac{10}{5} = 2 \quad \therefore \frac{n-r}{r+1} = 2 \quad \therefore n - 3r - 2 = 0 \quad \therefore n - 3r = 2$$

$$\frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{n - (r+2) + 1}{r+2} = \frac{14}{10} = \frac{7}{5} \quad \therefore \frac{n-r-1}{r+2} = \frac{7}{5} \quad \therefore 5n - 12r - 19 = 0$$

$$r = 3, n = 11$$

Q(18) Using the properties of the determinant prove that

$$\begin{vmatrix} Y+Z & X & X \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} = 2 \begin{vmatrix} 0 & Z & Y \\ Z & 0 & X \\ Y & X & 0 \end{vmatrix}$$

$$r_1 - (r_2 + r_3) \begin{vmatrix} 0 & -2Z & -2Y \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} = -2 \begin{vmatrix} 0 & Z & Y \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} \quad r_2 - r_1, r_3 - r_1$$

Q(19) Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8(1 - \sqrt{3}i)$ then write the solution set

$$X = 8, Y = -8\sqrt{3} \therefore r = \sqrt{8^2 + (-8\sqrt{3})^2} = 16$$

$$\tan \theta = \sqrt{3} \text{ in the 4th quadrant } \therefore \theta = -\frac{\pi}{3}$$

$$\therefore Z^4 = 16 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

$$\therefore Z = 2 \left(\cos \frac{-\frac{\pi}{3} + 2\pi r}{4} + i \sin \frac{-\frac{\pi}{3} + 2\pi r}{4} \right)$$

$$\text{When } r=0 \text{ then } Z_1 = 2 \left(\cos \frac{-\pi}{12} + i \sin \frac{-\pi}{12} \right) = 2e^{\frac{-\pi}{12}i}$$

$$\text{When } r=1 \text{ then } Z_2 = 2 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) = 2e^{\frac{5\pi}{12}i}$$

$$\text{When } r=-1 \text{ then } Z_3 = 2 \left(\cos \frac{-7\pi}{12} + i \sin \frac{-7\pi}{12} \right) = 2e^{\frac{-7\pi}{12}i}$$

$$\text{When } r=2 \text{ then } Z_4 = 2 \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right) = 2e^{\frac{11\pi}{12}i}$$

$$S.S = \{ 2e^{\frac{-\pi}{12}i}, 2e^{\frac{5\pi}{12}i}, 2e^{\frac{-7\pi}{12}i}, 2e^{\frac{11\pi}{12}i} \}$$

Q(20) $\left(\frac{a-dw}{aw^2-d} - w^2 \right)^2 = \dots\dots\dots$

$$\frac{aw^3 - dw}{aw^2 - d} - w^2 = \frac{w^2(aw^2 - d)}{(aw^2 - d)} - w^2 = (w^2 - w)^2 = (\pm\sqrt{3}i)^2 = -3$$

Q(21) Find the number of diagonals of a decagon? ${}^{10}C_2 - 10 = 35$

Q(22) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1 Z_2) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2} \right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$

$$\theta_1 + \theta_2 = \frac{5\pi}{18} \quad \text{and} \quad \theta_1 - \theta_2 = \frac{\pi}{9} \quad \therefore \theta_1 = 35^\circ = \frac{7\pi}{36}$$

(23) Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8(1 - \sqrt{3}i)$ then write the solution set

$$X = 8, Y = -8\sqrt{3} \quad \therefore r = \sqrt{8^2 + (-8\sqrt{3})^2} = 16$$

$$\tan \theta = \sqrt{3} \text{ in the 4th quadrant } \therefore \theta = -\frac{\pi}{3}$$

$$\therefore Z^4 = 16 \left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right) \right)$$

$$\therefore Z = 2 \left(\cos \frac{-\frac{\pi}{3} + 2\pi r}{4} + i \sin \frac{-\frac{\pi}{3} + 2\pi r}{4} \right)$$

$$\text{When } r=0 \text{ then } Z_1 = 2 \left(\cos \frac{-\pi}{12} + i \sin \frac{-\pi}{12} \right) = 2e^{\frac{-\pi}{12}i}$$

$$\text{When } r=1 \text{ then } Z_2 = 2 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) = 2e^{\frac{5\pi}{12}i}$$

$$\text{When } r=-1 \text{ then } Z_3 = 2 \left(\cos \frac{-7\pi}{12} + i \sin \frac{-7\pi}{12} \right) = 2e^{\frac{-7\pi}{12}i}$$

$$\text{When } r=2 \text{ then } Z_4 = 2 \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right) = 2e^{\frac{11\pi}{12}i}$$

$$S.S = \left\{ 2e^{\frac{-\pi}{12}i}, 2e^{\frac{5\pi}{12}i}, 2e^{\frac{-7\pi}{12}i}, 2e^{\frac{11\pi}{12}i} \right\}$$

Q(24) Using the properties of the determinant prove that $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = (x+2)(x-1)^2$

$$C_1 + C_2 + C_3 \therefore \begin{vmatrix} X+2 & 1 & 1 \\ X+2 & X & 1 \\ X+2 & 1 & X \end{vmatrix} = (X+2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & X & 1 \\ 1 & 1 & X \end{vmatrix} \quad r_2 - r_1, r_3 - r_1$$

$$(X+2) \begin{vmatrix} 1 & 1 & 1 \\ 0 & X-1 & 0 \\ 0 & 0 & X-1 \end{vmatrix} = (X+2)(X-1)^2$$

Q(25) If the number of terms in the expansion $(X + Y)^{2n-1}$ equals 12 terms then $n = \dots$

$$2n - 1 + 1 = 12 \quad \therefore 2n = 12 \quad \therefore n = 6$$

Q(26) If the side length of a triangle are $\frac{1}{2}|n|$, $|n - 2|$ and $|2 - n|$ cm

Then the numerical value of the area of the triangle =cm²

$$n - 2 \geq 0, 2 - n \geq 0 \quad \therefore n = 2 \text{ or } 3$$

$$\text{if } n = 2 \text{ sides are } 1, 1, 1 \text{ area} = \frac{1}{2} \times 1 \times 1 \times \sin 60^\circ = \frac{\sqrt{3}}{4}$$

$$\text{Q(27)} \left(\frac{3 + 5\omega}{5 + 3\omega^2} + \frac{5 + 3\omega^2}{3 + 5\omega} \right)^8 =$$

Q(28) Number of solution(natural) such that $a + b + c = 7$

$${}^{n+r-1}C_r = {}^{3+7-1}C_7 = 36 \quad (n) \text{ number of variables, } (r) \text{ their sum}$$

Similar example

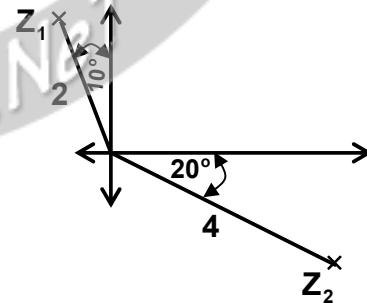
Number of ways to distribute 4 identical balls among 3 boxes

$${}^{n+r-1}C_r = {}^{3+4-1}C_3 \quad (n) \text{ number of boxes, } (r) \text{ number of balls}$$

Q(29) If Z_1, Z_2 , are two complex numbers represented On argend's plane as in the

opposite figure Find the form of $X+iY$ the number $\frac{Z_2}{Z_1}$

$$\frac{4(\cos 100^\circ + i \sin 100^\circ)}{2(\cos -20^\circ + i \sin -20^\circ)} = 2(\cos 120^\circ + i \sin 120^\circ) = 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \sqrt{3} - i$$



Q(30) The principle amplitude of the number $Z = 1 - i$ is

Q(31) Solve the following system of linear equations using the inverse matrix where $4X+Y=0$, $X+2Z=15$, $Y-7Z=0$

$$|A| = \begin{vmatrix} 4 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -7 \end{vmatrix} = -1 \quad \text{The cofactor matrix} \begin{pmatrix} -2 & 7 & 1 \\ 7 & -28 & -4 \\ 2 & -8 & -1 \end{pmatrix}$$

$$\therefore \text{Adj}(A) = \begin{pmatrix} -2 & 7 & 2 \\ 7 & -28 & -8 \\ 1 & -4 & -1 \end{pmatrix} \quad \therefore A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ -1 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ -1 & 4 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 15 \\ 0 \end{pmatrix} = \begin{pmatrix} -105 \\ 480 \\ 60 \end{pmatrix}$$

Q(32) Prove that: $\begin{vmatrix} X & a & a \\ a & X & a \\ a & a & X \end{vmatrix} = (X+2a)(X-a)^2$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} X+2a & a & a \\ X+2a & X & a \\ X+2a & a & X \end{vmatrix} = (X+2a) \begin{vmatrix} 1 & a & a \\ 1 & X & a \\ 1 & a & X \end{vmatrix} = (X+2a) \begin{vmatrix} 1 & a & a \\ 0 & X-a & 0 \\ 0 & 0 & X-a \end{vmatrix}$$

$$= (X+2a)(X-a)^2$$

Q(33) prove that $\begin{vmatrix} X+y+2 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$

$$= 2(X+Y+1)^3$$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} 2X+2Y+2 & X & Y \\ 2X+2Y+2 & 2X+Y+1 & Y \\ 2X+2Y+2 & X & X+2Y+1 \end{vmatrix} = (2X+2Y+2) \begin{vmatrix} 1 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$$

$r_2 - r_1, r_3 - r_1$

$$= 2(X+Y+1) \begin{vmatrix} 1 & X & Y \\ 0 & X+Y+1 & 0 \\ 0 & 0 & X+Y+1 \end{vmatrix} = 2(X+Y+1)^2$$

Q(34) In the expansion of $(X+Y)^n$ in descending power of X if T_2, T_3, T_4 are respectively 240, 720, 1080 evaluate the value of each of X, Y, n

Answer :

$$\frac{T_3}{T_2} = \frac{n-2+1}{2} \times \frac{Y}{X} = \frac{720}{240} = 3 \rightarrow (1) \quad , \quad \frac{T_4}{T_3} = \frac{n-3+1}{3} \times \frac{Y}{X} = \frac{1080}{720} = \frac{3}{2} \rightarrow (2)$$

$$\text{dividing 1,2} \therefore \frac{n-1}{2} \times \frac{3}{n-2} = 2 \quad \therefore \frac{3n-3}{2n-4} = 2 \quad \therefore 4n-8 = 3n-3 \quad \therefore n = 5$$

$$\therefore \text{from 1} \quad \therefore \frac{Y}{X} = \frac{3}{2} \quad \therefore Y = \frac{3X}{2}$$

$$T_2 = {}^5 C_1 \times \left(\frac{3X}{2}\right)^1 X^4 = 240 \quad \therefore X^5 = 32 \quad \therefore X = 2 \quad \therefore Y = 3$$

Q(35) If $X = \{A, B, C, D, E, F\}$ then The number of triangles whose vertices $\in X$ equals

① 15

② 20

③ 17

④ 21

$${}^6 C_3 = 20$$

Q(36) $(2 + 7\omega + 2\omega^2)(2 + 7\omega^2 + 2\omega^4) = \dots$

① 25ω

② 25

③ 125

④ 20

$$(-2\omega + 7\omega)(-2\omega^2 + 7\omega^2) = 5\omega \times 5\omega^2 = 25\omega^3 = 25$$

Q(37) If $A = \begin{pmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$ and If $\text{rank}(A) = 2$ then $m = \dots\dots$

$$\begin{vmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 0 \quad \therefore 4m = 8 \quad \therefore m = 2$$

Q(38) Put the number $Z = \frac{8}{1+\sqrt{3}i}$ in the trigonometric form

And hence find its two square roots in the exponential form

$$Z = \frac{8}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = 2 - 2\sqrt{3}i \quad \therefore r = 4 \quad 4^{\text{th}} \text{ quadrant}$$

$$\theta = -\tan^{-1} \sqrt{3} = -\frac{\pi}{3}$$

$$\sqrt{Z} = 2e^{\frac{-\pi+2K\pi}{2}} \quad \text{put } K=0, K=1$$

Q(39) Using matrix solve : $3X+Y-2Z=-3$, $2X+7Y+3Z=9$, $4X-3Y-Z=7$

$$|A| = \begin{vmatrix} 3 & 1 & -2 \\ 2 & 7 & 3 \\ 4 & -3 & -1 \end{vmatrix} = 88$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{88} \begin{pmatrix} 2 & 7 & 17 \\ 14 & 5 & -13 \\ -34 & 13 & 19 \end{pmatrix} \begin{pmatrix} -3 \\ 9 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Q(40) The amplitude of $(Z_1Z_2) = \frac{\pi}{6}$ and The amplitude of $(Z_1Z_3) = \frac{2\pi}{9}$

and The amplitude of $(Z_2Z_3) = \frac{5\pi}{18}$ then amplitude $(Z_1Z_2Z_3)$

$$\theta_1 + \theta_2 = 30^\circ, \theta_1 + \theta_3 = 40^\circ, \theta_2 + \theta_3 = 50^\circ$$

$$\therefore 2(\theta_1 + \theta_2 + \theta_3) = 120^\circ \quad \therefore \theta_1 + \theta_2 + \theta_3 = 60^\circ = \frac{\pi}{3}$$

Q(41) If $\begin{pmatrix} X-1 & 4 \\ 2 & X+1 \end{pmatrix}$ is singular matrix then $X=$

$$X^2 - 1 = 8 \quad \therefore X^2 = 9 \quad \therefore X = \pm 3$$

Q(42) If $(X-2) \times {}^n C_3 = {}^n P_3$ then $X=$

$$(X-2) = 6 \quad \therefore X = 8$$

Q(43) If 35, 21, 7 are the coefficient of three consecutive terms in the expansion $(1+X)^n$ find the value of n and the order of these terms

Let the terms are T_r, T_{r+1}, T_{r+2}

$$\frac{\text{coef} T_{r+1}}{\text{coef} T_r} = \frac{n-r+1}{r} \times \frac{1}{1} = \frac{21}{35} = \frac{3}{5} \rightarrow 5n - 8r + 5 = 0$$

$$\frac{\text{coef} T_{r+2}}{\text{coef} T_{r+1}} = \frac{n-(r+1)+1}{r+1} \times \frac{1}{1} = \frac{7}{21} = \frac{1}{3} \rightarrow 3n - 4r - 1 = 0$$

$$\therefore n = 7, r = 5$$

Q(44) Without expanding prove that $\begin{vmatrix} X & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x-a)(x-b)(x+a+b)$

$$C_1 + C_2 + C_3 \begin{vmatrix} X+a+b & a & b \\ X+a+b & x & b \\ X+a+b & b & x \end{vmatrix} = (x+a+b) \begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & b & x \end{vmatrix} = r_2 - r_1, r_3 - r_1$$

$$(x+a+b) \begin{vmatrix} 1 & a & b \\ 0 & X-a & 0 \\ 0 & b-a & X-b \end{vmatrix} \begin{vmatrix} 1 & a+b & b \\ 0 & X-a & 0 \\ 0 & 0 & X-b \end{vmatrix} = (x+a+b)(X-a)(X-b)$$

Q(45) without expanding the determinant prove that

$$\begin{vmatrix} a+b+2 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix} = 2(a+b+1)^3$$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} 2a+2b+2 & a & b \\ 2a+2b+2 & 2a+b+1 & b \\ 2a+2b+2 & a & a+2b+1 \end{vmatrix} = (2a+2b+2) \begin{vmatrix} 1 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix}$$

$$= 2(a+b+1) \begin{vmatrix} 1 & a & b \\ 0 & a+b+1 & 0 \\ 0 & 0 & a+b+1 \end{vmatrix} = 2(a+b+1)^3$$

Q(46) If $Z = \sin \frac{\pi}{9} + i \cos \frac{\pi}{9}$ find the cubic roots of $(\bar{Z})^9$

$$Z = \cos 70^\circ + i \sin 70^\circ$$

$$\bar{Z} = \cos 70^\circ - i \sin 70^\circ \therefore \bar{Z} = \cos -70^\circ + i \sin -70^\circ$$

$$\begin{aligned} (\bar{Z})^9 &= \cos -70^\circ \times 9 + i \sin -70^\circ \times 9 = \cos -630^\circ + i \sin -630^\circ \\ &= \cos 90^\circ + i \sin 90^\circ \end{aligned}$$

$$\sqrt[3]{} = \cos \frac{90^\circ + 2\pi r}{3} + i \sin \frac{90^\circ + 2\pi r}{3}$$

$$\text{When } r=0 \quad Z_1 = \cos 30^\circ + i \sin 30^\circ \quad e^{\frac{\pi i}{6}}$$

$$\text{When } r=1 \quad Z_2 = \cos 150^\circ + i \sin 150^\circ \quad e^{\frac{5\pi i}{6}}$$

$$\text{When } r=-1 \quad Z_2 = \cos -90^\circ + i \sin -90^\circ \quad e^{\frac{-\pi i}{2}}$$

Q(47) The coefficient of X^5 in the expansion $(3 - 2X)^7$ equals

$$T_{r+1} = {}^nC_r (-2X)^r (3)^{n-r} = {}^7C_r (-2X)^r (3)^{7-r} \quad \therefore r = 5$$

$$T_6 = {}^7C_5 (-2)^5 (3)^{7-5} = -6048$$

Q(48) find the two square roots of the complex number :

$Z = 2 - 2\sqrt{3}i$ in the trigonometric form

Q(49) If $Z = \sin 60^\circ - i \cos 60^\circ$, then the principle amplitude of $Z =$

Q(50) $(3 + 7\omega + 3\omega^2)(3 - 7\omega^2 + 3\omega)$

Q(51) The greatest value of $(3 + 2X)^6$ at $X=1$ is

$$\frac{T_{r+1}}{T_r} \geq 1 \quad \therefore \frac{6-r+1}{r} \geq 1 \quad \therefore 14 - 2r \geq 3r \quad \therefore r = 2$$

\therefore the greatest term is T_3

Q(1) The distance between the two planes

$3X+2Y-6Z-14=0$ and $3X+2Y-6Z+21=0$ isunits

$$\frac{|3(0)+2(7)-6(0)+21|}{\sqrt{3^2+2^2+6^2}} = 3 \text{ units}$$

Q(2) If the point $(-2,4,m)$ lies on the sphere

$(X+2)^2 + (Y-1)^2 + (Z-3)^2 = 25$ then one value of $m=....$

$$3^2 + (m-3)^2 = 25 \quad \therefore m-3 = 4 \quad \therefore m = 7$$

Q(3) If $\vec{A} = (4, -k, 6)$, $\vec{B} = (2, 2, m)$ and $\vec{A} \parallel \vec{B}$, then $k + m =$

$$\frac{4}{2} = \frac{-K}{2} = \frac{6}{m} \quad \therefore m = 3, K = -4 \quad \therefore K + m = -1$$

Q(4) Find the standard form and the general form of the equation of the plane passing through point $(3, -5, 2)$ and the vector $n = (2, 1, 1)$ is normal to the plane.

$$\boxed{n \cdot r = n \cdot A}$$

$$(2,1,1) \cdot (X,Y,Z) = (2,1,1) \cdot (3,-5,2)$$

$$2X + Y + Z = 2 \times 3 + 1 \times -5 + 1 \times 2 = 3 \quad \therefore 2X + Y + Z - 3 = 0$$

Q(5) If $A = (-3, 1, 2)$, $B = (3, 4, -1)$, find the area of the parallelogram in which A and B are two adjacent sides.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix} = -9\hat{i} + 3\hat{j} - 15\hat{k}$$

$$\therefore \|\vec{A} \times \vec{B}\| = \sqrt{(-9)^2 + (3)^2 + (-15)^2} = 3\sqrt{35} \text{ unit area}$$

Q(6) If the X axis cut the sphere which center $(3,-4,12)$ and its radius length 13cm at the two points A and B then AB equals

$$(X-3)^2 + 16 + 144 = 13^2 \quad \therefore (X-3)^2 = 9 \quad \therefore X = 0 \text{ or } X = 6$$

$$\therefore \text{the two points } A(0,0,0) \text{ and } B(6,0,0) \quad \therefore AB = 6$$

Q(7) Find the Cartesian equation of the plane
 $(X, Y, Z) = (2, 3, 5) + t_1(-1, 3, 4) + t_2(6, 1, -2)$ where t_1 and t_2 are parameters

$(-1, 3, 4)$, $(6, 1, -2)$ are the direction vectors of two lines in the plane

To get the normal to this plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 4 \\ 6 & 1 & -2 \end{vmatrix} = -10\hat{i} + 22\hat{j} - 19\hat{k}$$

$$\therefore (X, Y, Z) \cdot (-10, 22, -19) = (2, 3, 5) \cdot (-10, 22, -19)$$

$$10X - 22Y + 19Z = 49$$

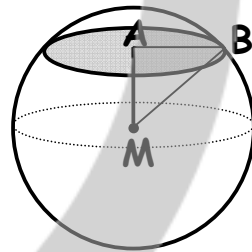
Q(8) If the vector A makes angles of measure α , β , θ with X, Y and Z axes then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \theta = \dots$

$$1 - \cos^2 \alpha + 1 - \cos^2 \beta + 1 - \cos^2 \theta = 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \theta) = 2$$

Q(9) If the plane $2x - 2y + z = 5$ intersect the sphere $(x - 2)^2 + (y - 3)^2 + (z + 2)^2 = 25$, then the area of the cross section (trace)

$$MA = \frac{|2(2) - 2(3) - 2 - 5|}{\sqrt{4 + 4 + 1}} = 3$$

$$\therefore AB = \sqrt{5^2 - 3^2} = 4 \quad \therefore \text{Area} = 16\pi$$



Q(10)

If θ is the measure of the angle included between

$A = (2, 0, 2)$, $B = (0, 0, 4)$, then $\theta = \dots$

$$\cos \theta = \frac{(2, 0, 2) \cdot (0, 0, 4)}{\sqrt{2^2 + 0^2 + 2^2} \times \sqrt{0^2 + 0^2 + 4^2}} = \frac{8}{8\sqrt{2}} = \therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

Q(11) If the vector A makes angles of measure α , β , θ with X, Y and Z axes then $\cos 2\alpha + \cos 2\beta + \cos 2\theta = \dots$

$$2\cos^2 \alpha - 1 + 2\cos^2 \beta - 1 + 2\cos^2 \theta - 1 = 2(\cos^2 \alpha + 2\cos^2 \beta + 2\cos^2 \theta) - 3 =$$

$$= 2 \times 1 - 3 = -1$$

Q(12) A line passing through the origin with the direction cosines $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ intersect the plane $3X+5Y+2Z-6=0$ at the point B find the length of OB

$$r = (0,0,0) + t(2,-3,6) = (2t,-3t,6t)$$

$$\therefore 3(2t) + 5(-3t) + 2(6t) - 6 = 0$$

$$6t - 15t + 12t - 6 = 0 \quad \therefore 3t = 6 \quad \therefore t = 2$$

$$\text{point of intersection(B)} = (4,-6,12)$$

$$OB = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$$

Q(13) Find the equation of the line of intersection of the two planes $X + 2Y - 2Z = 1$, $2X + Y - 3Z = 5$

First method:

$$-2X - 4Y + 4Z = -2$$

$$2X + Y - 3Z = 5$$

$$-3Y + Z = 3 \quad \text{put } Y = 0 \quad \therefore Z = 3 \quad \therefore X = 7 \quad \therefore P_1 = (7,0,3)$$

$$\text{Put } Z = 0 \quad \therefore Y = -1 \quad \therefore X = 3 \quad \therefore P_2 = (3,-1,0)$$

$$r = (7,0,3) - (3,-1,0) = (4,1,3)$$

$$\text{Equation : } (7,0,3) + t(4,1,3) \quad \therefore \frac{X-7}{4} = \frac{Y}{1} = \frac{Z-3}{3}$$

Q(14) If $L_1 : \frac{X+2}{-1} = \frac{Y+3}{3} = \frac{Z+5}{2}$ is perpendicular to the line

$L_2 : \frac{X}{2} = \frac{Y-5}{K} = \frac{Z-6}{m}$ then the value of $3K+2m=$

$$(-1,3,2) \cdot (2,K,m) = 0 \quad \therefore 3K + 2m = 2$$

Q(15)

If $\vec{A} = (1, -2, 1)$, $\vec{B} = (-2, 1, 2)$, then the component of \vec{A} in the direction of B =

$$\frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} = \frac{(1,-2,1) \cdot (-2,1,2)}{\sqrt{(-2)^2 + 1^2 + 2^2}} = -\frac{2}{3}$$

Q(16) If The two sphere $(X-3)^2 + Y^2 + (Z-3)^2 = 16$,
 $(X+1)^2 + (Y-4)^2 + (Z-K)^2 = 25$ Touch each other find the value of K

$$C_1 = (3,0,3) \quad C_2 = (-1,4,K) \quad r_1 = \sqrt{16} = 4 \quad r_2 = \sqrt{25} = 5$$

$$\therefore \sqrt{4^2 + 4^2 + (K-3)^2} = 5 + 4 = 9$$

$$\therefore \sqrt{32 + (K-3)^2} = 9$$

$$(K-3)^2 + 32 = 81 \quad \therefore (K-3)^2 = 49$$

$$K-3 = 7 \rightarrow K = 10 \quad \text{or} \quad K-3 = -7 \therefore K = -4$$

Q(17) Find the equation of the sphere has its center at the point $(5,-2,3)$ and touch the plane $3X+2Y+Z=0$

$$\text{Radius} = \frac{|3 \times 5 + 2 \times -2 + 3|}{\sqrt{3^2 + 2^2 + 1^2}} = \sqrt{14}$$

$$\text{Equation } (X-5)^2 + (Y+2)^2 + (Z-3)^2 = 14$$

Q(18) Find the equation of a plane which bisects perpendicularly the line joining the points $(2,3,4)$ and $(4,5,8)$

$$\text{Mid point } \left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2} \right) = (3,4,6)$$

$d = (4-2, 5-3, 8-4) = (2,2,4) = (1,1,2)$ is normal to the required plane

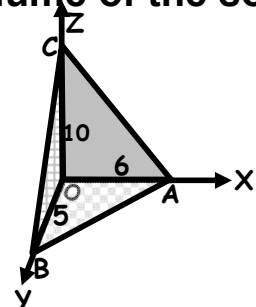
$$n \cdot r = n \cdot A \quad \therefore (1,1,2) \cdot (X,Y,Z) = (1,1,2) \cdot (3,4,6)$$

$$X + Y + 2Z = 3 + 4 + 12 = 19$$

Q(19) If the plane $10X + 12Y + 6Z = 60$ intersected with the axes X, Y and Z at the points A, B and C respectively then the volume of the solid ABCO where O is the origin point equalscube unit

$$\text{Volume of pyramid} = \frac{1}{3} \text{base area} \times \text{height}$$

$$= \frac{1}{3} \left(\frac{1}{2} \times 5 \times 6 \right) \times 10 = 50$$



Q(20) If $A=(2\cos\theta, \log_5 X, \sin\theta)$ and $A=(\cos\theta, \log_3 27, 2\sin\theta)$ and $A \cdot B = 11$
Then $X =$

$$2\cos^2\theta + \log_5 X \times \log_3 27 + 2\sin^2\theta = 11$$

$$\therefore 2\cos^2\theta + 2\sin^2\theta + \log_5 X \times \log_3 27 = 11 \quad \therefore 2 + \log_5 X \times \log_3 27 = 11$$

$$\log_5 X \times \log_3 27 = 9 \quad \therefore \log_5 X = 3 \quad \therefore X = 5^3 = 125$$

Q(21) The radius length of the sphere

$X^2 + Y^2 + Z^2 - 2X - 6Y + 10Z - 1 = 0$ equalslength unit

$$\text{Center} = (1, 3, -5) \quad \text{radius} = \sqrt{1^2 + 3^2 + (-5)^2 + 1} = 6$$

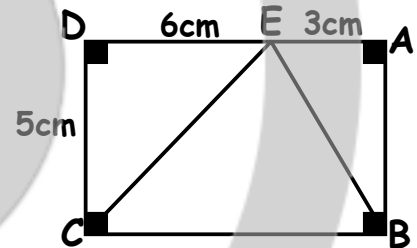
Q(22) In the opposite figure ABCD is a rectangle

$E \in \overline{AD}$ then $\vec{EB} \cdot \vec{EC} =$

$$C = (0, 0), \quad B = (9, 0), \quad E = (6, 5)$$

$$\vec{EB} = B - E = (3, -5) \quad \vec{EC} = C - E = (-6, -5)$$

$$\vec{EB} \cdot \vec{EC} = (3, -5) \cdot (-6, -5) = -18 + 25 = 7$$



Q(23)

If $\vec{A} = (2, 3, -4)$ and $\vec{B} = (4, 2, m)$ and $\vec{A} \perp \vec{B}$ then $m = \dots$

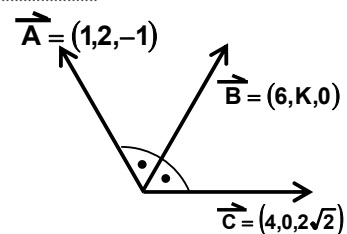
$$A \cdot B = 0 \quad \therefore 2 \times 4 + 3 \times 2 - 4m = 0 \quad \therefore 8 + 6 - 4m = 0$$

$$\therefore 14 - 4m = 0 \quad \therefore m = \frac{7}{2}$$

Q(24) In the opposite figure, the value of $k =$

$$(6, k, 0) \cdot (4, 0, 2\sqrt{2}) = (6, k, 0) \cdot (1, 2, -1)$$

$$24 = 6 + 2k \quad \therefore 2k = 18 \quad \therefore k = 9$$



Q(25) Find the standard form and the general form of the equation of the plane passing through point (3, -5, 2) and the vector $n = (2, 1, 1)$ is normal to the plane.

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{A}$$

$$(2,1,1) \cdot [(X, Y, Z) - (3, -5, 2)] = 0 \quad \therefore (2,1,1) \cdot (X - 3, Y + 5, Z - 2) = 0$$

$$2(X - 3) + 1(Y + 5) + 1(Z - 2) = 0 \quad \therefore 2X - 6 + Y + 5 + Z - 2 = 0$$

$$2X + Y + Z - 3 = 0$$

Q(26)

Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors $-12\mathbf{i} - 3\mathbf{k}$, $3\mathbf{j} - \mathbf{k}$ and $2\mathbf{i} + \mathbf{j} - 15\mathbf{k}$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

Q(27) Find the vector form of the equation of the straight line passing through point (3, -1, 0) and the vector (-2, 4, 3) is a direction vector for it.

$$\vec{r} = \text{point} + t (\text{direction vector})$$

① The vector equation:

$$\vec{r} = (3, -1, 0) + t(-2, 4, 3)$$

② Parametric equations:

$$(X, Y, Z) = (3, -1, 0) + t(-2, 4, 3)$$

$$\therefore X = 3 - 2t, \quad Y = -1 + 4t, \quad Z = 0 + 3t$$

③ The Cartesian equation:

$$\therefore t = \frac{X-3}{-2}, \quad t = \frac{Y+1}{4}, \quad t = \frac{Z}{3} \quad \therefore \frac{X-3}{-2} = \frac{Y+1}{4} = \frac{Z}{3}$$

Q(28) Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors $-12\hat{i} - 3\hat{k}$, $3\hat{j} - \hat{k}$ and $2\hat{i} + \hat{j} - 15\hat{k}$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

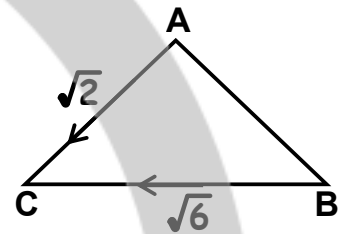
Q(29) In the given figure $\|BC\| = \sqrt{6}$ and $\|AC\| = \sqrt{2}$
 $BA = (-1, 0, 1)$ then $BA \cdot BC = \dots$

① 1

② 2

③ 4

④ 3



$$\therefore \|BA\| = \sqrt{1+0+1} = \sqrt{2} \therefore m(\angle A) = m(\angle B)$$

$$\cos(\angle B) = \cos(\angle C) = \frac{2+6-2}{2\sqrt{2} \times \sqrt{6}} = \frac{\sqrt{3}}{2} \therefore BA \cdot BC = \sqrt{2} \times \sqrt{6} \times \frac{\sqrt{3}}{2} = 3$$

Q(30) If $(X-2)^2 + (Y+4)^2 + (Z-2)^2 = 1$, $(X+4)^2 + (Y-4)^2 + (Z-2)^2 = 4$
 Are the equations of two spheres then the distance between their centers

$$\sqrt{(2+4)^2 + (-4-4)^2 + (2-2)^2} = 10$$

Q(31) Find the equation of the line passing through the point $(1, 2, 3)$ Perpendicular to the plane $2X - 3Y + Z + 1 = 0$

$$r = (1, 2, 3) + t(2, -3, 1)$$

Q(32) If The Y axis cut the circle which center $(3, -4, 12)$ and its radius 13cm at the points A and B then $AB = \dots$

$$(X-3)^2 + (Y+4)^2 + (Z-12)^2 = 13^2 \therefore (0-3)^2 + (Y+4)^2 + (0-12)^2 = 13^2$$

$$\therefore (Y+4)^2 = 16 \therefore Y+4 = \pm 4 \therefore Y = 0 \text{ or } Y = -8$$

$$A = (0, 0, 0) \text{ and } B = (0, -8, 0) \therefore AB = \sqrt{0+8^2+0} = 8$$

Q(33) The equation of the plane passing through the points (1,-2,5) and vector (2,1,3) is perpendicular to it is

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{A} \quad \therefore (2,1,3) \cdot (X,Y,Z) = (2,1,3) \cdot (1,-2,5)$$

$$\therefore 2X + Y + 3Z = 15$$

Q(34) The principle amplitude of the number $Z = 1 - i$ is

The number in 4th quadrant $\therefore \theta = -\tan^{-1} 1 = -\frac{\pi}{4}$

Q(35) The equation of the plane intercepting from coordinates axes(X,Y,Z) parts a,b,c respectively such that $ab=2$, $ac=3$, $bc=6$

$$a^2 b^2 c^2 = 36 \quad \therefore abc = 6 \quad \therefore \frac{abc}{ab} = c = 3, \quad \frac{abc}{ac} = b = 2, \quad \frac{abc}{bc} = a = 1$$

$$\frac{X}{1} + \frac{Y}{2} + \frac{Z}{3} = 1 \quad \therefore 6X + 3Y + 2Z - 6 = 0$$

Q(36) The general equation of the sphere whose centre (3,4,-5) and touch the YZ plane is ...

Q(37) If the two straight lines $L_1: X=2t-1, Y=t+1, Z=t-1$
 $L_2: X=at-1, Y=2t+1, Z=bt-2$ are parallel then $a+b=...$

$$\frac{a}{2} = \frac{1}{2} = \frac{1}{b} \quad \therefore a = 4, b = 2 \quad a + b = 6$$

Q(39) If the two planes $X-3Y+mZ=5$ and $3X+KY+6Z=10$ are parallel then $Km=.....$

$$\frac{1}{3} = \frac{-3}{K} = \frac{m}{6} \quad \therefore K = -9, m = 2 \quad \therefore mK = -18$$

Q(39) Find the equation of the plane passing through the point (2,1,4) and perpendicular to each of the two planes $7X+Y+2Z=6$, $3X+5Y-6Z=8$

\therefore required plane \perp to the planes

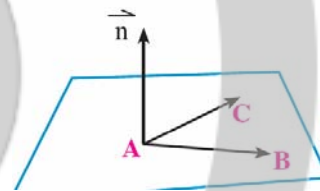
\therefore (7,1,2) , (3,5,-6) are direction vectors lies in the required plane

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16\hat{i} + 48\hat{j} + 32\hat{k} \quad \div 16 \quad \therefore \text{required (n) is } (1,-3,-2)$$

$$r \cdot (1,-3,-2) = (1,-3,-2) \cdot (2,1,4)$$

Q(40) Find the equation of the plane passing through points (3, -1, 0) , (2, 1, 4) and (0, 3, 3).

$$n = AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{vmatrix} = -10\hat{i} - 9\hat{j} + 2\hat{k}$$



$$n \cdot r = n \cdot A$$

$$(-10,-9,2) \cdot r = (-10,-9,2) \cdot (3,-1,0) = -21$$

$$-10X - 9Y + 2Z + 21 = 0$$

Q(41) Find the equation of the line of intersection of the two planes $X + 2Y - 2Z = 1$, $2X + Y - 3Z = 5$

$$-2X - 4Y + 4Z = -2$$

$$2X + Y - 3Z = 5$$

$$-3Y + Z = 3 \quad \text{put } Y = 0 \quad \therefore Z = 3 \quad \therefore X = 7 \quad \therefore P_1 = (7,0,3)$$

$$\text{Put } Z = 0 \quad \therefore Y = -1 \quad \therefore X = 3 \quad \therefore P_2 = (3,-1,0)$$

$$r = (7,0,3) - (3,-1,0) = (4,1,3)$$

$$\text{Equation : } (7,0,3) + t(4,1,3) \quad \therefore \frac{X-7}{4} = \frac{Y}{1} = \frac{Z-3}{3}$$

Q(42) If the plane $20X + 15Y + 12Z = 60$ intersected with the axes X, Y and Z at the points A, B and C respectively then the volume of the solid ABCO where O is the origin point equalscube unit

$$\frac{1}{3} \times \left(\frac{1}{2} \times 3 \times 4 \right) \times 5 = 10$$

Q(43) If the plane $\frac{X}{4} + \frac{Y}{2} + \frac{Z}{2} = 1$ cut the coordinates axes at A and B and C then the area of ΔABC equal

① 12

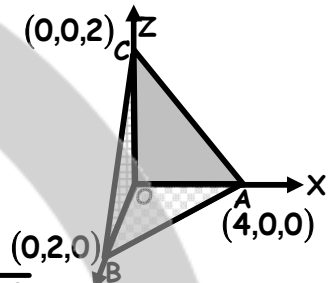
② 10

③ 6

④ 4

$$AC = C - A = (-4, 0, 2), \quad AB = B - A = (-4, 2, 0)$$

$$A = \frac{1}{2} AC \times AB = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & 2 \\ -4 & 2 & 0 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 8\hat{k} \therefore \text{Area} = \frac{1}{2} \sqrt{4^2 + 8^2 + 8^2} = 6$$



Q(44) A sphere of center $(2, -1, -2)$ and radius 3 units placed on the plane $2X + 6Y - 3Z + K = 0$ then $K = \dots\dots\dots$

$$\frac{2(2) + 6(-1) - 3(-2) + K}{\sqrt{4 + 36 + 9}} = 3 \therefore \frac{4 - 6 + 6 + K}{7} = 3 \therefore 4 + K = 21 \therefore K = 21$$

Q(45) Measure of the angle between the two planes $X + Y - 1 = 0$ and $Y + Z - 1 = 0$ equals $^\circ$

$$\text{Angle between the normal } \cos \theta = \frac{(1, 1, 0) \cdot (0, 1, 1)}{\sqrt{1^2 + 1^2 + 0} \sqrt{0^2 + 1^2 + 1^2}} = \frac{1}{2} \therefore \theta = 60^\circ$$

Q(46) The equation of the sphere its center $M(1, -2, -5)$ and touch the XY plane

$$(X - 1)^2 + (Y + 2)^2 + (Z + 5)^2 = 25$$

Q(47) Find the measure of the angle between the two straight lines :

$$r_1 = (2, -1, 3) + t_1(-2, 0, 2) \quad \text{and} \quad r_2 : X = 1, \frac{Y-4}{3} = \frac{Z+5}{-3}$$

$$d_1 = (-2, 0, 2) \quad , \quad d_2 = (0, 3, -3)$$

$$\cos \theta = \frac{d_1 \cdot d_2}{\|d_1\| \times \|d_2\|} = \frac{(-2, 0, 2) \cdot (0, 3, -3)}{\sqrt{(-2)^2 + 0^2 + 2^2} \sqrt{0^2 + 3^2 + (-3)^2}} = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1} \frac{1}{2} = 60^\circ$$

Q(48) A sphere of centre (1,2,1) touches the plane $X+Y+Z=1$ find the equation of the sphere

Radius of the sphere is the perpendicular length from the centre
To the plane

$$= \frac{|1+2+1-1|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}}$$

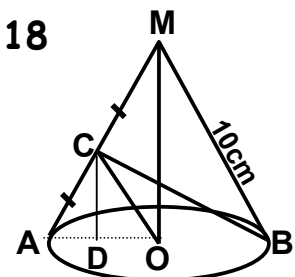
Equation of the sphere

$$(X-1)^2 + (Y-2)^2 + (Z-1)^2 = 3$$

Q(49) In the opposite figure, a right circular cone, the perimeter of its base = 12π cm C is the midpoint of AM then $OC \cdot OA =$

$$2\pi r = 12\pi \quad \therefore r = 6 \quad OA = 6 \quad \therefore MO = 8 \text{ cm} \quad , \quad CO = 5 \quad \therefore DO = 3 \text{ cm}$$

$$CD = 4 \text{ cm} \quad \therefore OC \cdot OA = \|OC\| \times \|OA\| \cos(\angle COA) = 5 \times 6 \times \frac{3}{5} = 18$$



Q(50) If $\|A\| = 2$, $\|B\| = 3$, $\|C\| = 12$ and A , B , C are mutually orthogonal then $\|A + B + C\| =$

$$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$$

$$= 4 + 9 + 144 = 157 + 0 + 0 + 0 = 157 \quad \therefore \|A + B + C\| = \sqrt{157}$$

Q(51) If $X = \frac{-1 - \sqrt{3}i}{2}$, $i^2 = -1$, then the numerical value of

$$X^8 + X^4 + 5 = \quad (4)$$

Q(52) Find the equation of the straight line passing through the point $(2, -1, 3)$ and intersects the straight line $r_1 = (1, -1, 2) + t(2, 2, -1)$ orthogonally.

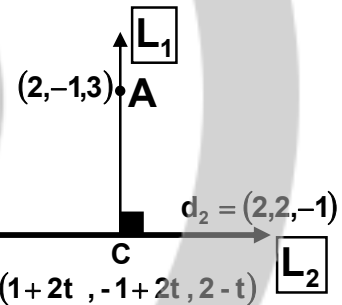
$$d_1 = AC = C - A = (2t - 1, 2t, -1 - t)$$

$$\therefore L_1 \perp L_2 \quad \therefore d_1 \cdot d_2 = 0$$

$$(2t - 1, 2t, -1 - t) \cdot (2, 2, -1) = 0$$

$$4t - 2 + 4t + 1 + t = 0 \quad \therefore 9t - 1 = 0 \quad \therefore t = \frac{1}{9}$$

$$\therefore d_1 = \left(-\frac{7}{9}, \frac{2}{9}, -\frac{10}{9}\right) = (-7, 2, -10)$$



Equation of the line is $r = (2, -1, 3) + t(-7, 2, -10)$

Q(53) Find the equation of the straight line passing through the point $(3, -1, 0)$ and intersect the straight line $r = (2, 1, 1) + t(1, 2, -1)$ orthogonally

$$(1, 2, -1) \cdot (-1 + t, 2 + 2t, 1 - t) = 0$$

$$\therefore t = -\frac{1}{3} \quad \therefore r = (3, -1, 0) + t(-1, 1, 1)$$

Q(54) Use the multiplicative inverse to solve the set of following equations $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z} = 1$, $\frac{1}{X} - \frac{1}{Y} + \frac{2}{Z} = \frac{1}{2}$, $\frac{2}{X} + \frac{3}{Y} - \frac{4}{Z} = \frac{4}{3}$

Where X and Y and Z not equals zeros

$$\begin{pmatrix} \frac{1}{X} \\ \frac{1}{Y} \\ \frac{1}{Z} \end{pmatrix} = \frac{1}{11} \begin{pmatrix} -2 & 7 & 3 \\ 8 & -6 & -1 \\ 5 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{4}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{pmatrix} \therefore \frac{1}{X} = \frac{1}{2} \therefore X = 2$$

Q(55) Find the equation of a sphere its diameter \overline{AB} where $A=(-1,5,4)$, $B=(5,1,-2)$

$$\text{center} = \left(\frac{-1+5}{2}, \frac{5+1}{2}, \frac{4-2}{2} \right) = (2,3,1)$$

$$\text{Radius} = \sqrt{(2+1)^2 + (3-5)^2 + (1-4)^2} = \sqrt{22}$$

$$\text{The equation is } (X-2)^2 + (Y-3)^2 + (Z-1)^2 = 22$$

Q(56) Find the measure of the angle between the two straight lines

$$r_1 = (2, -1, 3) + t_1(-2, 0, 2) \quad \text{and} \quad X = 1, \frac{Y-4}{3} = \frac{Z+5}{-3}$$

$$d_1 = (-2, 0, 2), \quad d_2 = (0, 3, -3)$$

$$\therefore \cos \theta = \frac{d_1 \cdot d_2}{\|d_1\| \times \|d_2\|} = \frac{(-2, 0, 2) \cdot (0, 3, -3)}{\sqrt{(-2)^2 + 0^2 + 2^2} \times \sqrt{0^2 + 3^2 + (-3)^2}} = \frac{1}{2} \therefore \theta = 60^\circ$$

